

1994

Analysis of the effect of in-class writing on the learning of function concepts in college algebra

Bernadette Marie Baker
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function concepts in college algebra**

Baker, Bernadette Marie, Ph.D.

Iowa State University, 1994

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**300 N. Zeeb Rd.
Ann Arbor, MI 48106**

Analysis of the effect of in-class writing on the learning
of function concepts in college algebra

by

Bernadette Marie Baker

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Department: Curriculum and Instruction
Major: Education (Curriculum and Instructional Technology)

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For the Department and Education Major

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For the Graduate College

Iowa State University
Ames, Iowa

1994

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To my husband, Bill,
and our children,
Bridget and Eric

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ABSTRACT

This study examined the effects of in-class writing to learn mathematics on college students in a college algebra course. The students in the two experimental groups wrote explanatory responses to prompts regarding a topic discussed in class the previous day twice a week for eight weeks. The teacher read and responded to all writing assignments the next day. The two control groups spent the time discussing additional examples as a class. All other aspects of the course were held constant for the 209 students in the study.

The first goal of the study was to investigate the effect of the in-class writing on mathematics achievement. The second goal was to investigate the effect of the in-class writing on the students' attitude toward mathematics. A third goal was to investigate whether the in-class writing treatment would be differentially effective for some students more than others based on previous mathematics achievement, length of time since the last math class, or self reported study habits.

Findings of the study showed that the in-class writing treatment was differentially effective on the attitudes of low achievement students. The low achievement writing students had significantly more positive attitudes toward mathematics at the end of the study than the low achievement nonwriting students. Also there was a significant interaction for the treatment x time since the last mathematics class. The students in the writing group who had not taken a math class for 1.5 years or more had significantly better achievement scores than the those in the control group who had not recently completed a math course. These two findings may have practical implications for making mathematics accessible to

more students through the use of writing assignments. In general, the study did not find a significant effect for either attitude or achievement for the writing groups.

GENERAL INTRODUCTION

Many articles have been written recently about the use of writing to learn mathematics (for example, Havens, 1990; Freeman & Murphy, 1990, 1992; Keith, 1988). Additionally, two recent books have included essays on using writing in teaching mathematics (Sterrett, 1990; Connolly & Vilardi, 1989). The articles and essays are overwhelmingly positive about the potential benefits of the use of writing but relatively little research has been reported on this subject (Smith et al., 1992). Writing to learn mathematics appeared to be a fertile area for further exploration. Thus the subject of this dissertation was a study of the use of in-class writing to learn mathematics in a college algebra course. The questions posed in this study dealt with the effects of writing on student achievement and attitude.

This dissertation included two papers to be submitted to scholarly journals. The first paper was a review of the articles and research that have been published about writing to learn mathematics. The second paper described the investigation of the use of in-class writing on the achievement and attitudes of college students enrolled in college algebra and reported the results of the study.

Dissertation Organization

This dissertation was written in a format that allows for the inclusion of papers to be submitted to scholarly journals. This is done in lieu of the chapter format but includes the same content. The references cited in each paper are included in a bibliography section following the general conclusion.

WRITING TO LEARN MATHEMATICS: A REVIEW
OF RESEARCH AND PRACTICE

A paper to be submitted to *Review of Educational Research*

Bernadette M. Baker

Introduction

Writing to learn in mathematics offers new promise in mathematics classrooms to improve learning. Writing about a topic forces the student to think about the topic, compare facts about relevant ideas, and synthesize (Emig, 1977). Despite the fact that some mathematics teachers feel poorly prepared to incorporate writing in their mathematics classes (Rose, 1989; Sterrett, 1990; McIntosh, 1991), many teachers are experimenting with writing assignments, some in controlled experiments and others more casually. Since the publication of the NCTM *Curriculum and Evaluation Standards for School Mathematics*, in 1989, with the inclusion of the communication standard, the interest in writing in mathematics has intensified (Carton, 1990; McGehe, 1991). The purpose of this article is to examine the uses that mathematics teachers are making of writing activities and to review research results concerning writing activities to learn mathematics. Directions for further research will be suggested.

The mathematics education community received the latest wake-up call for reforming the teaching and learning of mathematics at all levels with the publication in 1989 of the *Curriculum and Evaluation Standards for School*

Mathematics by the National Council of Teachers of Mathematics. Incorporating writing into the learning of mathematics may be useful in implementing the reform. In too many classrooms, students memorize formulae for various mathematical situations but do not understand the significance of those symbols (Hurwitz, 1990). Writing in mathematics can help to change the focus from computation and manipulation of symbols to processing and interpretation of the symbols and strategies (LeGere, 1991). A number of national reports have documented the failure of current approaches to produce students who understand and can use the mathematics they have studied. In *Everybody Counts*, the National Research Council (1989) stated:

Research in learning shows that students actually construct their own understanding based on new experiences that enlarge the intellectual framework in which ideas can be created. Consequently, each individual's knowledge of mathematics is uniquely personal.

Mathematics becomes useful to a student only when it has been developed through a personal intellectual engagement that creates new understanding. Much of the failure in school mathematics is due to a tradition of teaching that is inappropriate to the way most students learn.

(p. 6)

One of the ways that some authors suggest that knowledge is constructed is through writing about what one is learning (Flower, 1985, Hayes, 1989, Smith, 1982, Kenyon, 1989). The *Curriculum and Evaluation Standards for School Mathematics*, in Standard 2 for the secondary (9-12) curriculum, calls for students to be able to think about and clarify their understanding of mathematical concepts and relationships and express mathematical knowledge orally and in writing

(NCTM, 1989). Using this perspective on learning changes the classroom environment to one where students experience mathematics as a creative activity rather than memorization of rote algorithms (Rose, 1989; LeGere, 1991).

Talking and writing usually occur in mathematics classrooms although frequently it is the teacher who is talking and most of the writing is of mathematical symbols (Connolly, 1989). Actually, most students already write relatively frequently in mathematics classes. They take notes, complete homework assignments, fill in worksheets, and work exercise sets. But much of the writing involves performing calculations or manipulating mathematical symbols. Little time is typically spent composing sentences or putting sentences together into paragraphs (Rose, 1989).

Writing has value both for students and for the teacher. For students, explaining orally or in writing how a problem was solved is useful both to the person explaining in clarifying her or his own thinking and for other students in gaining new insight from a different perspective on the problem (NCTM, 1989). Additionally, writing permits active participation by students who are less confident with their speaking skills (McGehe, 1991), and participation by all students at the same time (Geeslin, 1977). For the teacher, writing can serve a diagnostic purpose, in identifying what students understand and where misconceptions have occurred (Birken, 1989; Keith, 1988).

In the next section, the relationship between concept development and writing will be explored from the perspective of process writing research and psychology with an effort made to show how writing can enhance concept development. Practitioner articles demonstrating the many ways that secondary and college mathematics instructors are using writing in their classrooms will be

reviewed. In addition, more formal research studies will be reviewed. Finally some conclusions and directions for additional research will be offered.

Concept Development and Writing

To learn, one needs to classify past experience and to fit the new experiences into these classes (Skemp, 1987). Much of this happens on an unconscious level (Flower, 1985). Acquisition of subtle concepts is the result of long term accumulation and sorting of experiences (Skemp, 1987). When concepts are related to each other, and linked into new concepts, this new structure is called a schema. Writing can be used as a means of organizing one's ideas, and fitting new information into existing schemata (Skemp, 1987). In an analogous explanation of the use of writing for learning, Emig (1977) theorized that in order to write about a concept, the student must classify the idea in terms of related and similar or dissimilar ideas. These ideas may be visual, verbal or a combination of the two. Writing requires work in translating between the various representations that the student carries (Emig, 1977). As a student writes, the words provide immediate feedback to the learner for review as to whether the meaning the student has developed at this time is conveyed by the words that are written. Also, because writing results in a product, a student can, as her or his understanding develops, look back at the words later to see if they still convey the meaning that the student now has constructed (Emig, 1977). Whereas only one student at a time may talk to a teacher, all the students in the class can write simultaneously and the teacher may provide feedback, also in writing, to each student (Geeslin, 1977). This ongoing dialogue is useful both to the student as understanding of concepts

develops and changes and to the teacher in planning classroom activities and discourse (Nahrgang & Petersen, 1986).

Several researchers of the writing process have proposed that writing is a problem solving activity (Flower, 1985; Hayes, 1989; Goodkin, 1982). Various strategies for process writing have been proposed. Flower (1985) suggested that useful writing strategies were brainstorming, WIRMI (what I really mean is), using notation techniques, and satisficing. Notation helps a writer to focus on the idea and its relationship to other ideas. Satisficing is a technique of getting some approximation of the idea down and waiting for a rewrite to get the perfect phrasing. The problem solving approach concentrated the writer's attention on the process, on what the writer wanted to say or get accomplished not on the final product (Flower, 1985) and promoted speculative thinking to solve problems and weigh consequences (Goodkin, 1982). Flower (1985) stated that "Writing is a powerful way to think problems through, because it helps you describe and name the conflicting parts of your own thinking." (p. 23). This use of verbal language is also the most available medium for students to use in learning (Emig, 1977). As noted by Skemp (1987), this naming of experiences is concept formation. By experiencing examples, and eventually nonexamples, one forms the concept of yellow or line. While one is not usually aware of all this while trying to write, the writer is planning, evaluating already expressed prose, detecting errors, setting goals, drawing inferences, trying new relationships or looking for patterns, and creating trial explanations (Flower, 1985). All of these activities contribute to concept learning.

Sometimes students propose that they understand a concept, but can't explain it (Flower, 1985). Using a constructivist perspective, Flower (1985)

theorized that new information is stored in mental representations in order to be remembered and used in thinking, but that these representations may be either verbal or nonverbal. When students can't express an idea that they believe they understand it is because they are struggling to translate a nonverbal representation into a verbal one (Flower, 1985). Writing requires one to be explicit and pick out the most pertinent aspects of the idea to express. This struggle to put ideas into words is a way of testing the ideas and one's knowledge of them (Smith, 1982; Flower, 1985). A similar commentary on the difficulty of assimilating new information into one's existing understanding is made by Skemp (1987): "We are not in a position to say what we mean by understanding. *To understand something means to assimilate it into an appropriate schema.* This explains the subjective nature of understanding and also makes clear that this is not usually an all-or-nothing state" (p. 29).

Most theorists identify three to five steps in the problem solving process. Probably the best known are Polya's (1957) four steps: understand the problem, devise a plan, carry out the plan, and look back at the solution. However Hayes (1989) identifies five problem solving steps: representation, planning, execution, evaluation, and consolidation. The three steps most commonly identified in the writing process are: planning, sentence generation, and revision (Hayes, 1989). The first two problem solving steps, representation and planning correspond to the planning process in writing. At that stage, the writer identifies the task to be accomplished (representation), and a sequence of steps to follow to get the task completed (planning). The sentence generation step is execution of the problem solving plan. Revision requires evaluation (did this writing accomplish my goal?) and consolidation (seeing new or different relationships between the ideas

explored in the writing) (Hayes, 1989). In an ethnographic study, this role of writing in abstracting and making connections was confirmed experimentally by analysis of writing samples and interviews (Goodkin, 1982). Since new information combines with existing information and is stored in long term memory in this new form, learning is taking place (Kenyon, 1989).

As one acquires new information, it is not usually checked carefully against existing information (Hayes, 1989). As noted by Skemp (1987), as well as Hayes (1989), this can lead to conflicting ideas. That is, one can adopt conflicting ideas but if these two conflicting ideas are not thought about together, one may not be aware of this incongruity (Hayes, 1989). The opportunity to write gives the learner the chance to relate ideas that have not been considered together before. This relating of ideas is conceptual learning.

A somewhat different perspective is that learning is the natural activity of the brain that takes place constantly whether we are aware of it or not (Smith, 1982). According to this theory, it is not memory capacity that causes problems in learning but relevance. Three conditions are necessary for learning: demonstration, engagement, and sensitivity (Smith, 1982). Besides the obvious meaning of demonstration, it can also be a demonstration of something to oneself, when one notices a relationship or imagines something, that is also a demonstration (Smith, 1982). Frequently in the writing process, one becomes aware of new relationships between concepts. The second condition of learning is engagement, which happens whenever one is involved in the learning situation. Certainly writing requires engagement or no sentences are generated. The third condition is sensitivity, or what the brain is interested in learning. Smith (1982) would assert that one sorts out one's learning by writing. This would correspond to Hayes'

(1989) notion of consolidation, that one tries out new relationships by writing about them.

Functions of Writing

In this section, since writing has long been a common activity in education, the types of writing that have been used for learning are explained. Three functions of writing in school have been identified: transactional, poetic, and expressive (Britton et al., 1975). The researchers defined transactional writing as writing to persuade or inform. In many classrooms, nearly all of the writing was transactional, such as term papers, essays, lab reports or book reviews. Assessment of learning was usually the purpose of this type of writing. A second type, poetic, was usually thought of as creative writing, and little use of this form was made outside of classes whose focus was creative writing. The third type of writing was called expressive and included writing whose purpose was exploring relationships and understanding, but not evaluation of the learning. Expressive writing activities included journal writing, problem solving and problem posing, explanations of errors or algorithms, some types of microthemes, letter writing, and freewriting. Whereas transactional writing was geared toward an audience, expressive writing was personal and did not necessarily have an audience other than the writer or a close friend. Thus expressive writing was more casual in form. Another difference between transactional and expressive writing was that expressive writing usually had a single purpose whereas transactional writing may serve more than one purpose. Expressive writing was usually used only to promote learning and understanding in a particular discipline (writing in the content

area) as opposed to transactional writing which may be used in the same way or may be used for the general purpose of improving the writing skills of the students (writing across the curriculum) (Miller, 1991). The next sections summarize the ways that transactional and expressive writing have been used to learn mathematics.

Using Writing to Teach Secondary Mathematics

Several types of writing assignments have been proposed for secondary mathematics students: composing word problems, rewriting a page of a student's textbook that she/he finds confusing, essay questions on tests, book reports, and research papers on topics at an appropriate level, such as biographies of famous mathematicians or outlines of the evolution of a particular area of mathematics (Johnson, 1983). Other ideas for writing assignments include concept maps and guided response writing (McGehe, 1991), study cards (Whitesitt, 1990), and journals (Watson, 1980; Nahrgang & Petersen, 1986; Hendel, 1993).

Journals

Journals are a form of expressive writing including entries about struggles in learning mathematics, feelings about their learning, efforts to rephrase text sections or major ideas in their own words, and continuing dialogue between the student and teacher. Journals are used for either in-class or out-of-class writing or both. In some implementations the students write in response to specific teacher prompts (Nahrgang & Petersen, 1986) and in others the writing topics are exclusively of the student's choosing, or a combination of the two (Watson, 1980; Hendel, 1993).

Sometimes the amount of time spent writing is specified (Nahrgang & Petersen, 1986), or the length of the desired writing, or the frequency of the writing (Hendel, 1993). Teachers reported that student responses became better organized and easier to read after students had been involved in writing for a period of time (Nahrgang & Petersen, 1986). The teachers read and responded to journal entries and both the students and the teachers felt that this activity was very beneficial for both the class and the teacher (Nahrgang & Petersen, 1986; Watson, 1980; Hendel, 1993). The students seemed to have gained some insight into their struggles in math and the teacher reported that writing was motivational to many of the students in advanced algebra (Watson, 1980). Teachers believed that they knew more about what their classes were learning and having problems learning, so the journals also served a diagnostic purpose (Nahrgang & Petersen, 1986; Watson, 1980). When informed of the journal writing activity at parent teacher conferences, the parents were very supportive and more students completed journal assignments in the following weeks (Hendel, 1993).

Problem Posing

Several practitioner articles reported classroom activities which involved students in writing problems (Woodward and Byrd, 1984; Carton, 1990; McGehe, 1991). Woodward and Byrd (1984) had eighth grade students write problems to fit a given graph. Students had to think carefully about the information provided in the graph and the authors reported that the students responded enthusiastically. In another middle school application, collaborative learning was paired with a writing activity having groups of four students write a set of a dozen problems (at the rate of about two a day) and prepare sample solutions for each (Carton, 1990). The

directions included proportions of the problems that must be related to specific content (fractions, geometry, percentages, etc.) as well as proportions for process and difficulty (one-step, two-step, easy, medium, difficult). The groups each completed a worksheet each day specifying the project topic, group members, a summary of the day's accomplishments, and a check sheet concerning member participation in the work, and the checking of results and language for accuracy and clarity (Carton, 1990). Problems could then be shared between groups, with other classes at the same level within the school, or by use of telecommunications with distant classes. The teacher reported very positive results for this project in a classroom including some special-needs students (Carton, 1990). In a third classroom, the students individually wrote and illustrated their word problems for classmates with both guided directions (write a story problem involving multiplication and subtraction) or more open-ended prompts (look at the picture at the front of the room and write a word problem to do with it) (McGehe, 1991). This activity was motivational and allowed students to bring their creativity to the their mathematics learning, in fact, eventually these problems were published by the classroom teacher (McGehe, 1991).

Explanations

Writing assignments using explanations have taken several forms: student authored manuals (Hurwitz, 1990), study cards (Whitesitt, 1990), concept maps and guided response writing (McGehe, 1991), letters (Havens, 1989), and explanations of processes (Havens, 1989; Evans, 1984). Sometimes these assignments were completed in class and at other times outside of class. Generally these were relatively short, informal writings although unlike journals they were usually

intended for an audience. Preparation of study cards including the mathematical term on one side of a 3x5 card and the definition and an example on the other side required students to identify important concepts and provided an efficient study review for tests (Whitesitt, 1990). They also provided a diagnostic tool for the teacher in identifying misconceptions.

In a general mathematics class, the students wrote explanations of an algorithm, applications of concepts, an advertisement, or the meaning of new vocabulary terms about twice a week in class (Havens, 1989). After an unsuccessful start to writing with these students due to their inadequate explanations of the mathematics, overwhelming grammatical problems, and failure to follow directions, the teacher incorporated peer conferences into the writing assignments. A class checklist to be completed in each peer conference included (1) the specific writing theme, (2) specification of format such as letter, summary, word problem, etc., (3) the required length such as one sentence, one paragraph, etc., and (4) one rule of sentence structure in addition to correct spelling (Havens, 1989). After the writing was completed in class, the students met in pairs with a peer and completed the checklist for each author's paper. The students then used the comments from the peer conference to revise their writing at home and turn it in at the next class period (Havens, 1989). This introduction of peer conferences improved the level of writing and the focus by the students on the specific prompt (Havens, 1989). Since these students had not been very successful previously in learning mathematics, the teacher was especially pleased that both she and the students agreed that these writing assignments were successful in expanding the students' abilities to learn mathematics (Havens, 1989).

In her fifth grade class, Evans (1984) used three types of writing assignments with her students, using another fifth grade class who did no writing as a control. Although the treatment class scored lower on the pretest for both of the units in the study (multiplication and geometry), the treatment class scored as well or better than the control class on the posttest measure (Evans, 1984). The researcher used three types of writing with her students: explanation of a process, such as how to perform multiplication with a zero as one of the digits in the multiplier, definitions of mathematical terms in the students own words, and explanations of errors that occurred on their papers.

In using concept maps, the students were not composing but did need to analyze the relationships among different parts of a concept in order to relate them (McGehe, 1991). In one middle school implementation of concept maps, the teacher named a concept and as students identified related responses, the teacher organized them on the blackboard into unnamed categories and asked the student to name the categories (McGehe, 1991).

Student authored texts

In the last several weeks of the semester, after months of complaints about the poor quality of the explanations in their mathematics text, students were given a chance to write a substitute text in their own words (Hendel, 1993). The course topics were distributed among the students honoring topic preferences when possible. Each student shared her/his rough draft with two other students in the class who made suggestions for improvements before the final version was submitted. This activity proved highly motivational for the students (Hendel, 1993).

Using Writing to Teach College Mathematics

Many of the same techniques such as journals (Marwine, 1989; Hartz, 1990; Talman, 1990; Britton, 1990; Freeman & Murphy, 1990) and explanations (Keith & Keith, 1985; Keith, 1988; Meyer, 1991; Goldberg, 1983; Socha, 1989; and LaGere, 1991) have been reported for using writing to teach college mathematics. Some additional uses of writing in college classes include microthemes (Martin, 1989), research papers (Snow, 1990; Freeman & Murphy, 1992; Stoughton, 1990; Goldberg, 1990; McDonald and Mett, 1990), lab reports (Gopen & Smith, 1989), and essays (Birken, 1990; Rauff, 1991; Snow, 1990; and Freeman & Murphy, 1992). The next several sections will detail the uses of these techniques in college teaching.

Journals

One use of journal writing was based on the philosophy that teaching is communication of meaning from one who knows about a certain discipline to someone who does not (Marwine, 1989). Journal writing was used to monitor the progress of understanding as it was occurring in the learning process. Marwine (1989) believed this was more fruitful for the learner than the "method of anticipation" (p.59) whereby the teacher attempted to anticipate and redirect the errors of understanding that one "thought" their students might succumb to. The teacher responded to all writing, and sometimes posed additional questions to students in his responses to stimulate further thinking. These students often shared their writing with each other in small groups which promoted their respect for their fellow students as partners in the learning endeavor (Marwine, 1989).

Students also used their journals as "a tool for learning" (Marwine, 1989, p. 65) in responding to texts and for writing about process: How is a problem like or unlike another, what strategy will be tried first and why? This teacher modeled journal writing for the first week or two but after that, writing took only five to ten minutes per period. Although he graded the journals weekly, Marwine was careful to be encouraging and not to correct ideas that students were just beginning to explore.

Other authors have used journals in other ways, for instance, to write weekly abstracts of the material discussed in class the previous week (Hartz, 1990). In a service course for social science majors, one of the effects was that it required students to keep up with their review of class notes and assignments. Like in Marwine's (1989) class, the journals were graded for effort and completeness but not for mathematical correctness or grammar since the purpose was to encourage the students to struggle to express the mathematics they were studying in their own words (Hartz, 1990). At the beginning of the term, questions were suggested for consideration to help the students get started in writing about mathematics (Hartz, 1990). A variation of the review writing was used in a trigonometry class where the students wrote an explanation of each day's homework assignment to someone who knows no trigonometry but wants to learn and then exchanged journals the next day in class and critiqued each other's explanations (Freeman & Murphy, 1990). The purpose of this journal activity was to ensure that students read the text before trying the homework assignments. The instructors found that both the style and the correct use of mathematical language improved as the semester progressed (Freeman & Murphy, 1990).

One controlled study involved the use of journals. In her high school geometry classes, Linn (1987) found that 95% of her students in the journal writing

classes *believed* that the journals increased their understanding of the geometry they were studying. Moreover, achievement scores did rise for the treatment classes. The researcher found that, at least while writing, the students became active learners because the writing forced them to synthesize the the information they were writing about (Linn, 1987). This writing brought more clearly into focus for the students both what they did and did not understand. Additionally, the journals served as a diagnostic tool for the instructor and opened up a different line of communication between the students and the instructor. Another journal investigation was a case study of one student's journal and freewriting in a college class in developmental mathematics (Powell & Lopez, 1989). In the case of this single student, an analysis of his writings showed that both his learning of mathematics and his attitude toward his ability to learn mathematics improved.

Several benefits to journal writing that have been identified were: students asked questions in their journals that they would not have asked in class or bothered to come to office hours for, journal writing allowed students to relate mathematics to their own disciplines, it made all students more active participants in their learning, especially the quieter students (Hartz, 1990), journal writing allowed students who missed class to test their understanding of the material they missed, students kept up with content better, and it allowed taking attendance without bothering to do so in class (Britton, 1990). Also, journals promoted getting the better students to process at a higher level, and can be used to involve students in self evaluation (Talman, 1990). For many students the use of a journal forces exploration of mathematics which is a new experience (Talman, 1990) . Some authors suggested that an example of what you expect, either something another

student wrote or one you made up yourself was very helpful at the beginning of the journal writing process (Talman, 1990; Britton, 1990).

Disadvantages to using a journal were the time necessary to read and respond to the journals, including the fact that only one or two small classes per semester could be involved, and the difficulty of judging the writing (Talman, 1990). Yet instructors, being well educated, did recognize when undergraduates were writing clearly and as mathematicians knew when they were relating mathematical ideas reasonably well (Talman, 1990). While formal writing is difficult for many students, the informal style of journal entries allowed students to explore half-understood ideas, ask questions, and maintain regular informal communication that was more comfortable for some students than personal conversations (Burkam, 1990).

Microthemes

Usually a single type written page, short focused writing assignments have been dubbed microthemes. A couple of authors described their use in science courses at the college level and suggested that these uses of writing may be helpful in other science and mathematics courses (Martin, 1989; Mullin, 1989). In a general education course in biology, students were permitted to use these writing assignments to lessen the value of test scores on their final course grade. The instructor believed that these writing assignments for students who may not be good test takers provided another way to demonstrate a firm grasp of their biology knowledge (Martin, 1989). Additionally, through students sharing their understandings, the instructor's own imagery of many biology concepts has been broadened and she felt that she knew these students much better than those who

did not complete writing assignments (Martin, 1989). The writing was not graded for grammar or style but only for the correctness of biology concepts expressed.

In both general education physics courses and advanced physics courses students wrote intuitive explanations of why a particular formula works or gave an analogy for explaining a physical phenomenon, such as why an airplane flies (Mullin, 1989). The instructor felt that this intuitive level was the one most physicists used in communicating with other professionals in the field but often did not share with their students. His purpose in assigning microthemes was to help students increase their understanding of the physics concepts underlying the various formulas that they learned. While the formal mathematical arguments were necessary for completeness and rigor, Mullin (1989) argued that the oral tradition of intuitive arguments was what produced understanding and that students needed to practice the art of developing analogies to explain the concepts underlying the mathematical formulae they had learned to manipulate.

A single study involving the use of microthemes in statistics was identified. In nine classes of undergraduate statistics, Smith, Miller, and Robertson (1992) tested the use of writing assignments to improve student learning and attitudes toward statistics. Each of the four instructors taught at least one writing and one nonwriting section, and efforts were made to hold all other aspects of the course constant. The writing assignments were responses to writing prompts and had to be typed on a single 3x5 index card. The small space for writing was designed to encourage a large amount of thinking followed by a small amount of writing. Approximately 5% of the semester grade was based on the writing assignments which were graded for completion but not correctness. A feedback sheet for statistical correctness and clarity of expression was used. The study did not find

statistically different achievement results for the writing and nonwriting groups as measured by final exam performance but did find significant differences in attitudes toward statistics. The attitude changes favored the writing students. A majority of the writing students judged that the microthemes helped both in learning the concepts and in communicating them.

Research Papers

Papers requiring students to complete library research or use other sources outside of the class notes and text were popular, especially in upper division mathematics courses (Snow, 1990; Stoughton, 1990; Burkam, 1990; Goldberg, 1990; Millman, 1990; McDonald & Mett, 1990; Kiltinen & Mansfield, 1990; Freeman & Murphy, 1992). These research papers were expository assignments, and some instructors made the papers semester long projects (McDonald & Mett, 1990; Goldberg, 1990; Kiltinen & Mansfield, 1990) while others limited the students to working for a specific time period (Stoughton, 1990; Millman, 1990; Goldberg, 1990). Most instructors allowed students to choose their own topics within some specified bounds but one instructor assigned topics to his students (Stoughton, 1990). It was generally agreed that some attention to helping the students identify a reasonable and difficulty-appropriate topic pays off as well in better papers. Many of the instructors also required an initial draft to be turned in for comment but not graded and found that this technique improved the quality of the papers considerably (Burkam, 1990; McDonald & Mett, 1990; Kiltinen & Mansfield, 1990; Snow, 1990; Freeman & Murphy, 1992; Millman, 1990).

Writing a research paper helped to broaden the student's view of the discipline of mathematics (Snow, 1990; Freeman & Murphy, 1992). Assignment of

a longer paper near the end of the semester gave the student the opportunity to explore an interesting topic in more depth (Burkam, 1990, Goldberg, 1990). Also, when students saw that great mathematicians also struggled for understanding it was very motivating for them and nonmajors came to greater appreciation of mathematics as a field.(Snow, 1990). A history of mathematics instructor required a research paper that included both mathematics and history concerning a topic discussed in the course (Goldberg, 1990). Another instructor assigned the first paper on a topic from a previous mathematics course that the student already understood well so that she/he concentrated on trying to learn about writing mathematics (Shoughton, 1990).

In an upper level statistics class, the students identified a problem on campus or within the community that could be solved using operations research techniques (McDonald & Mett, 1990). They spent the semester researching the problem, writing monthly versions of a prospectus which included their latest findings and plans for the next step, and a final written and oral report of a proposed solution to the problem. Although both the teacher and the students found this type of assignment had advantages in a small advanced class, it required a big time commitment from both the teacher and the students (McDonald & Mett, 1990).

A use of research papers with peer review in an abstract algebra class was described by Kiltinen & Mansfield (1990) . During the semester, two of the problem sets assigned were to be submitted as both written reports and mathematical proofs. A "writing fellow", a student identified both as a good mathematics student and a good writer worked with the students. They handed in a first draft of the paper to the "fellow" for feedback in an individual conference and then handed in

both the first and the final drafts to the instructor for a grade (Kiltinen & Mansfield, 1990). The instructor prepared a set of written guidelines for the students addressing the issues of statement of the problem, audience, grammar, the use of diagrams, and use of the mathematical term "clearly", among other topics. As others, Kiltinen believed that this technical writing helped the students to more clearly understand the mathematics they were studying. From the perspective of the writing fellow: "When I receive a set of rough drafts, it is apparent who knows what they're talking about and who doesn't. A lot of symbols on a piece of paper can look convincing - a lot of words can't. The words reveal themselves." (Mansfield in Kiltinen and Mansfield, p. 95)

Explanations

Numerous examples of the use of explanatory writing in college mathematics classrooms can be found (Keith & Keith, 1985; LeGere, 1991; Socha, 1989; Keith, 1988; Goldberg, 1983; Meyer, 1991; Birken, 1989; Hayden, 1990; Snow, 1990). Like journals, the purpose of explanatory writing was for exploration of understanding. Explanations were usually short and informal in style, with the emphasis on the clarity of thinking rather than the grammatical structure. In their college classes, Keith & Keith (1985) found that writing assignments provided learning opportunities for all students, both those strong and weak in mathematics. The writings stimulated meaningful class discussions as well because they made all the students active participants in their learning (Keith & Keith, 1985; LeGere, 1991). One of the most popular types of assignments was on-the-spot assessments of students' understanding of a concept (Keith & Keith, 1985; LeGere, 1991; Meyer, 1991; Birken, 1989) or anticipatory overnight assignments that

concerned a concept to be discussed the next day in class (Keith & Keith, 1985). The emphasis was on the more informal expressive writing assignments because the purpose of this writing was learning as opposed to assessment for making a judgment on how much was learned in order to assign a grade.

A controlled study on teaching college mathematics using explanatory writing was reported (Youngberg, 1988). The students were enrolled in four sections of an elementary algebra course, where classes engaged in writing for the last ten minutes of class each day about that day's or the previous day's topic except when a test was scheduled. After the first assignment, the students directed their writing to their classmates and the classmates provided a response after which the instructor responded to both the original writing and the response. The investigator measured student achievement on each of five tests during the semester. On the final exam, the writing group mean was significantly different than the control group, with the writing class outperforming the control group. Also she found that the writing assignments had a positive impact on achievement for those concepts that were directly related to the writing assignments. Unlike Evans (1984) who found the greatest gains for the weakest students, in Youngberg's study, the positive effect was greater for the better students. Overall, the students were neutral about the writing experience with some believing it benefited them and others expressing dislike for the activity. The classes were quite small with 56 total students in the study.

Two investigators have used written explanations of the steps in solving different kinds of problems in teaching remedial mathematics. In her freshman course, Fundamentals of Mathematics, Pallmann (1983) paired with a composition teacher in having the treatment students write detailed explanations of the

processes followed in solving different kinds of arithmetic and elementary algebra problems. The composition teacher included assignments asking the students to explore their feelings about learning mathematics, the relevance of mathematics to their lives, and to pose word problems based on their knowledge of mathematics. The difference in achievement was not significantly different for the experimental and control groups but the retention rate for completing the course was significantly different.: 87% of the treatment group students completed the remedial course while only 39% of the control group was still in the course at completion. The results were considered favorable for the treatment since most of the weaker students were retained in the course while most of the weaker control students dropped the course.

Another study involving writing solution steps for remedial classes was reported by Lesnak. In his basic algebra classes at the college level, Lesnak (1989) designed an experiment to test the effect on achievement of using writing to learn activities with his classes. Using two control and two treatment classes, he found that the writing classes performed significantly better than the non writing classes. The student writing consisted mostly of carefully writing the steps to complete in order to solve each type of problem that the students were studying and writing explanations of corrections to problems done incorrectly. The writing students finished the course with a significantly higher average than the control students (Lesnak, 1989). In addition to the quantitative results, changes in attitude were also reported (Lesnak, 1989). The investigator did not measure these changes quantitatively but felt that they were probably the more important result of his study. Early in the course, the hostility of the writing students bordered on rebellion and by the end, all the 52 writing students assessed the value of the

writing positively, including eight students who did not pass the course (Lesnak, 1989). The students reported that the writing helped them to prepare for tests, to identify the material they did not understand, and increased their confidence in their ability to learn algebra. The investigator believed that the writing activities helped to provide a bridge between mathematical reasoning which his students believed they could not engage in and verbal reasoning which they did engage in already.

Additional examples of exploratory writing assignments reported included summaries of a concept and visual image translation (describe this graph so a friend you are talking to on the telephone could visualize it) (Keith, 1988). Writing a synopsis of the strategy for solving a problem, stating a definition, algorithm, or theorem, or inventing a problem were other assignments used successfully by Keith (1988). Personal math histories, and analysis of the solution of a problem were incorporated into writing assignments (LeGere, 1991) as were summary sheets (to prepare for a quiz or test) and analysis of the adequacy of a summary sheet after the exam (Meyer, 1991).

Informal explanation of a problem solution, and a translation activity from symbols to words was used successfully by Birken (1989) who believed that she was much more clear about the thinking of her students, what they understood and where their thinking was incorrect, and that she was better able to redirect incorrect thinking. Within the homework assignments, Snow (1990) required that certain problems, in addition to being solved also had to be explained in words. Students found this writing provided a helpful review guide for later study since the formulae and symbols after a few days or weeks passed had less meaning for the student (Snow, 1990). A final example of the variety of uses of explanatory writings was a

class log assignment whose purpose was to provide a concise written record of what occurred each day in a class (Soucha, 1989). The duty for compiling the daily log rotated among members of the class and included class notes, examples and diagrams, copies of class handouts, and homework assignments. The teacher used the class log as a barometer of class understanding (Soucha, 1989).

In an experiment with students in fourth through twelfth grades, the students were asked to write about various probability concepts (Geeslin, 1977). The investigator found that few of the students could express complete or correct mathematical statements. A rationale for using writing in the classroom both as a learning device for the student and as a diagnostic tool for the teacher was stated. The researcher believed that writing assignments should be very short at first and could be an explanation of the meaning of a single mathematical word (Geeslin, 1977). When some progress in expressing mathematical ideas has been developed, the task could be expanded to comparing or contrasting a pair of math terms. It was believed that the ability to write about mathematical ideas would improve the students' ability to discuss these ideas in group problem-solving situations (Geeslin, 1977).

A comparison of error patterns in college students studying elementary algebra was reported by Gordon (1988). The purpose of the study was to examine whether having students write about the work required in solving three kinds of algebraic fraction problems rather than working additional examples would have an effect on their error patterns on quizzes. The results were inconclusive about whether differences that occurred were due to the writing treatment.

Essays

Essays assigned in mathematics classes were generally somewhat longer and more formally written than the expressive writing assignments but were less formal and shorter than research papers. Several practitioners described their use of essay questions on mathematics exams to test understanding (Hayden, 1990; Freeman & Murphy, 1992; Snow, 1990; Birken, 1989). Even when announced before the exam, such questions were often a surprise to students. Frequently the students had little idea how to organize their thoughts for such an answer. Having students practice such exercises before the exam and receive feedback or sharing with the students a sample of what one considered an adequate answer improved the prospects of getting reasonable answers (Hayden, 1990). This author felt the use of writing helped to focus his teaching away from computational and manipulative skill and onto the meaning of the computation or manipulation and the statistical terms being studied (Hayden, 1990).

Other types of essays assigned included those on enrichment topics (Goldberg, 1990), compare and contrast statements, or argument of a position on an "if, then" statement (Snow, 1990). Letters to a younger student on a topic that the student was familiar with but had not mastered were used as a take home quiz by Meyer (1991). One of the interesting things she noticed about the letters was how uniformly supportive they were of the younger students' ability to master the topic described. Essays based on journal articles or reports on films viewed in the Resource Center were assigned by Goldberg (1990) based on her suggestions of both appropriate journals and specific journal articles and films. In an unusual use of essays, Rauff (1990) asked an upper division class to choose seven topics from the course and develop a prose version of a cognitive map at the end of the course

explaining not only the concepts but also their relation to each other. The students found this both challenging and very rewarding. This same instructor engaged another upper division class in poetic writing by requiring them to write a piece of mathematical fiction as an integrative mathematical thinking exercise (Rauff, 1991).

Several suggestions about essay writing that emerged from the articles were related to audience. One should be specific about the audience for the essay and if possible to make it someone other than the teacher. Since students know that the teacher already knows the topic, they tend to write less clearly and completely than if the audience is another student in the class who was ill or a younger sibling who may not have already studied this subject (Snow, 1990). Other suggestions included giving frameworks about the desired length and whether or not the use of outside sources was expected.

Lab Reports

Weekly lab reports were paired with the teaching of the "reader expectation" theory of writing in calculus classes taught by Gopen and Smith (1989). Reader expectation theory is used to help writers structure their exposition to be better understood by the reader by placing key components of the substance in certain well-defined places in the structure of the prose. Once the students gained some skill in this method of writing, each student shared a first draft with a peer who used the technique to give initial feedback to the writer. This was additionally seen to be a learning experience for both the reader and the writer (Gopen and Smith, 1989). Then, after revision, the student turned in a second draft to the teacher who gave feedback, mostly on substance, since the paper has already been revised once. This version, usually one to three pages of expository writing, was graded. The

authors claimed that they had incorporated writing assignments into their calculus classes with substantial success and without excessive extra effort for the instructors. They believed that thought and expression of thought were so closely intertwined that one can not be skilled in one without skill on the other (Gopen and Smith, 1989).

Discussion

Many practitioners and some researchers report benefits in the use of writing to learn mathematics. What was most striking in reading these many articles was the often repeated statement that teachers found out so much about student thinking by reading their writing. Certainly for teachers to know more about what and how students are thinking is an important contribution that writing makes. Clear presentations of mathematical information is important in the mathematics classroom, but is only the beginning of learning; writing allows for the processing of that information (LeGere, 1991).

The dialogue between teacher and student is important for several reasons: one can interact with more students by responding to their writing than would be possible in class and this personalizes the learning environment (Miller, 1991). For a student to learn an algorithm, she needs to be able to explain it to herself, but without writing rarely gets to explain it to the teacher (Keith & Keith, 1985). This rehearsal can be an important part of the learning process, and provides for teacher intervention in case of misunderstanding at a crucial point in the process. Actually, this rehearsal is similar to the benefits in understanding that accrue from teaching someone else something: your own understanding increases (Goldberg,

1983). Further, this communication provides an informal means of assessment to the teacher about how and how much of a lesson was understood (Miller, 1991). And writing assignments can help meet the needs of students at all levels of ability because it is challenging even to strong students (Keith & Keith, 1985). Finally, writing provides another way for the teacher to learn about her students as people (McIntosh, 1991) and can provide a renewal of teaching energy and freshness of attitude to veteran teachers (LeGere, 1991).

Another common thread was the belief that most teachers and some students shared about the usefulness of the writing experiences. Perhaps this signals some educationally significant event is occurring in writing although one may not be sure of what it is or how to measure it. Most reports concluded that writing assignments were motivational for students. With a goal in the *Standards* to have all students, indeed society at large, value mathematics more, writing to learn mathematics may help students to achieve this if they believe that writing is helping them learn. In fact, Evans (1984) found that the students with the lowest achievement at the beginning of her study made the greatest gains so writing may help make mathematics learning accessible to more students. Writing generally requires more precision in thinking than talking because one needs more organization of one's thoughts in writing (Geeslin, 1977). However the additional thinking and organization of thoughts will still not produce research level writing so realistic expectations are important (Johnson, 1983). After writing about a concept, more students will have ideas to contribute to class discussions of that concept (Keith & Keith, 1985).

Certainly the question of whether appropriate writing activities in mathematics can be designed has been answered in the affirmative. Many

interesting and motivational assignments have been suggested in these articles. The issue of time for writing in a full curriculum has also been creatively modeled by many practitioners, including using small blocks of class time and out of class writing assignments. Some authors have decided that less is more and simply cover less content but at a greater depth through using writing, while others substitute some writing assignments for homework exercises, or combine writing assignments with collaborative learning activities (LeGere, 1991).

The issue of grading or evaluating the writing has been handled in a number of different ways. While some concern over the additional burden of reading the writing that one asks students to do is valid, several models for peer feedback and check-sheets for teacher response have been reported and show promise of faster ways to respond to the writing. Some teachers give credit for completion of the writing, or make it a part of the participation grade (McIntosh, 1991). Other teachers use a holistic rating scale that includes just plus, minus or a check mark (for ok) (Freeman & Murphy, 1990). On some assignments, the teacher may have a check sheet of specific things that will be evaluated (Burton, 1985, Havens, 1989). Several authors used peer evaluation of the first draft or prior to the teacher's evaluation (Keith, 1988; Youngberg, 1989; Gopen & Smith, 1989). It is important to be clear both to oneself and to one's students on what the purpose of the writing is: Is it writing to learn or writing to show learning? These obvious differences require different types of evaluation. In using writing to learn mathematics in her classroom, Keith (1988) found that control of grammar and mechanics were so closely related to mathematical understanding that grading for grammar was not an issue. From the perspective of writing to show mathematical learning, Henriksen

(1990) refused to read any assignment that was not written in complete sentences. The paper was returned ungraded for corrections.

Another theme that seemed to weave through these articles was a sense that classrooms using writing were intense, interactive classrooms where both the teacher and the student were actively involved in the learning process. Teachers who used writing no longer subscribed to the transmission of knowledge model of teaching, but were interacting with their students as the students were constructing their mathematical knowledge. The students were accepting responsibility for their learning and were partners in it, rather than seeing themselves as recipients of the teacher's knowledge. In this way, writing was empowering to both students and teachers in helping them to become collaborators in the education process.

Several authors referred to frustration on the part of both the teachers and the students at the beginning of trying to use writing to learn mathematics (Lesnak, 1989; Geeslin, 1977; Havens, 1989). Some of this was due to students' lack of previous opportunities to write in mathematics, while other problems were caused by poorly designed assignments or lack of specificity in the assignment. But these were problems that can be solved, and many articles contained useful ideas for getting started. In general, the suggestions were to start slowly, with one class, be as specific as possible with the assignment and the intended audience. Be clear about the purpose of the writing: for learning or to show learning. Work to develop an atmosphere of trust and respect with the students so that students will feel comfortable in accepting the new challenge that writing in mathematics was for most of them. Be specific in how the writing will be evaluated. Some teachers give an example of an acceptable answer for the first assignment. If the assignment is a

long term one, intermediate deadlines are helpful in producing a better set of papers.

Conclusions

Many practitioners are experimenting with writing assignments in mathematics classes but too little research has been published for conclusive evidence about what writing may contribute to the learning of mathematics. Some of the positive research results have involved remedial mathematics students and if writing proves to be especially beneficial to these students, it would be one step in making more mathematics accessible to all students. Additional studies examining the effects of writing on special populations will help to identify students who might benefit from the use of writing to learn mathematics. What is evident is that many teachers and students *believe* that writing contributes to greater understanding of mathematics and more positive attitudes. Further research will be needed to confirm or dispel these beliefs. Practitioners would benefit from studies that measure the effect of short writing assignments since this type could more easily be accommodated into the current system of teaching. Additional research on the value and efficacy of peer feedback for learning would also provide needed information to teachers. Use of telecommunications applications for real but remote audiences should be explored as another method of response to student writing as well.

ANALYSIS OF THE EFFECT OF IN-CLASS WRITING ON THE LEARNING OF FUNCTION CONCEPTS IN COLLEGE ALGEBRA

A paper to be submitted to the *Journal for Research in Mathematics Education*

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Abstract

This study examined the effects of in-class writing to learn mathematics in 209 college students in a college algebra course. The students in the two experimental sections wrote explanatory responses to teacher prompts while the control groups discussed additional examples as a class. The goals of the study were to investigate the effects of in-class writing on mathematics achievement and on student attitudes towards mathematics. A third goal was to investigate whether the writing treatment was differentially effective for some students based on previous mathematics achievement, length of time since the last mathematics class, or self-reported study habits. Findings showed that the treatment was differentially effective on the attitudes of low achievement students and that there was a significant interaction between treatment and time since the last mathematics class.

Purpose and Rationale

Many teachers have experimented with writing in mathematics classrooms and felt that it offered great promise as a learning tool (for example, Johnson, 1983;

Miller, 1991; Geeslin, 1977; Rose, 1989; Hendel, 1993; Birken, 1989; Nahrgang & Petersen, 1986), but few controlled studies have been reported (Youngberg, 1989; Evans, 1984; Pallmann, 1983; Lesnak, 1989). Interest in writing in mathematics first emerged from the writing to learn and writing across the curriculum innovations in the late 1970s and early 1980s. More recently, since the publication of the NCTM *Curriculum and Evaluation Standards for School Mathematics* in 1989, with the inclusion of the communication standard, the interest in writing in mathematics has been intensified (Carton, 1990; McGehe, 1991). The purpose of this article was to report on an investigation of in-class writing in college algebra at a midwestern university. The goals of the study were to examine the effects of the writing treatment on the students' achievement and attitudes towards mathematics. Additionally, the data were analyzed to determine if the writing treatment was differentially effective for some students based on their past mathematics achievement, length of time since the last mathematics class, or self-reported study habits.

Writing to learn

One way that learning has been understood was as a sorting of experiences into classes (Skemp, 1987). Acquisition of subtle concepts was the result of long term accumulation and sorting of these experiences (Skemp, 1987). These classes of experiences or mental representations have been called schemata (Skemp, 1987) and were sometimes verbal and sometimes nonverbal (Flower, 1985). Writing about an idea has been proposed as one help to sorting out one's thoughts about it and to test one's knowledge of the idea (Smith, 1982; Flower, 1985).

The act of writing has been described as a problem solving process (Hayes, 1989; Flower, 1985). As such, the focus of the writing was the thinking and writing process rather than the product and the process was recursive, meaning that it did not necessarily proceed on a linear path from step to step (Hayes, 1989).

Recursive processes can be interrupted by another part of the process repeatedly. Hayes (1989) named five steps in problem solving (representation, planning, execution, evaluation, and consolidation) and described how these steps corresponded to the steps in the writing process. The planning process in writing matched the planning and representation steps in problem solving, where the task was identified and planning on how to accomplish the task occurred. Secondly, when the schemata were translated into sentences, this was the execution step. Revision of the sentences occurred during the evaluation and consolidation stage (Hayes, 1989). Here especially the recursive nature of writing was obvious because the revision and evaluation occurred during the sentence generation stage as well as after it.

Writing can have different formats as well as different functions. Three functions of writing in school have been identified: transactional, poetic, and expressive (Britton et al., 1975). The researchers defined transactional writing as writing to persuade or inform. In many classrooms, nearly all of the writing was transactional, such as term papers, essays, lab reports or book reviews. Assessment of learning was usually the purpose of this type of writing. A second type, poetic, was usually thought of as creative writing, and little use of this form was made outside of classes whose focus was creative writing. The third type of writing was called expressive and included writing whose purpose was exploring relationships and understanding, but not evaluation of the learning. Expressive

writing activities included journal writing, problem solving and problem posing, explanations of errors or algorithms, some types of microthemes, letter writing, and freewriting. Whereas transactional writing was geared toward an audience, expressive writing was personal and did not necessarily have an audience other than the writer or a close friend. Thus expressive writing was more casual in form. Another difference between transactional and expressive writing was that expressive writing usually had a single purpose whereas transactional writing sometimes served more than one purpose. Expressive writing was usually used only to promote learning and understanding in a particular discipline (writing in the content area) as opposed to transactional writing which was used in the same way or for the general purpose of improving the writing skills of the students (writing across the curriculum) (Miller, 1991). The next section summarized the ways that expressive writing has been used to learn mathematics.

Writing to learn mathematics

Writing assignments in secondary school using explanations have taken several forms: student authored manuals (Hurwitz, 1990; Hendel, 1993), study cards (Whitesitt, 1990), concept maps and guided response writing (McGehe, 1991), and letters and explanations of processes (Havens, 1989). Sometimes these assignments are completed in class and at other times outside of class. Generally these are relatively short, informal writings although unlike journals they are usually intended for an audience. They may also provide a diagnostic tool for the teacher in identifying misunderstandings of mathematical concepts (Nahrgang & Petersen, 1986; Birken, 1989).

Numerous examples of the use of explanatory writing in college mathematics classrooms were found (Keith & Keith, 1985; LeGere, 1991; Socha, 1989; Keith, 1988; Goldberg, 1983; Meyer, 1991; Birken, 1989; Hayden, 1990; Snow, 1990). Explanations were usually short and informal in style, with the emphasis on the clarity of thinking rather than the grammatical structure. In their college classes, Keith & Keith (1985) found that writing assignments provided learning opportunities for all students, because even good students found them challenging. The writings stimulated meaningful class discussions as well because they made all the students active participants in their learning (Keith & Keith, 1985; LeGere, 1991).

One of the most popular types of assignments was on-the-spot assessments of students' understanding of a concept (Keith & Keith, 1985; LeGere, 1991; Meyer, 1991; Birken, 1989) or anticipatory overnight assignments that concerned a concept to be discussed the next day in class (Keith & Keith, 1985). The emphasis was on the more informal expressive writing assignments because the purpose of this writing was learning as opposed to assessment for making a judgment on how much was learned in order to assign a grade. The current study used this format as well.

Informal explanation of a problem solution, and a translation activity from symbols to words was used for greater clarity about the thinking of the students, what they understood and where their thinking was incorrect (Birken, 1989). This enabled the instructor to be better able to redirect incorrect thinking. In another use of writing, within the homework assignments certain problems had to be explained in words in addition to being solved (Snow, 1990). Students found this writing provided a helpful study guide for review since the formulae and symbols a few

days or weeks later had less meaning for the student than their explanations (Snow, 1990).

Most of the controlled research that has been conducted has involved expressive writing, usually with the students involved in explaining a process, a concept, a definition, or how two ideas are similar and dissimilar (Evans, 1984; Gordon, 1988; Pallmann, 1983; Lesnak, 1989; Youngberg, 1989). Most of these studies found results favoring the writing treatment as is detailed below.

Explanatory writing assignments were used by Evans (1984) with her fifth grade class, and another fifth grade class who did no writing served as a control. Although the treatment class scored lower on the pretest for both of the units in the study (multiplication and geometry), the treatment class scored as well or better than the control class on the posttest measures (Evans, 1984). Three types of writing were used in the study: explanation of a process, such as how to perform multiplication with a zero digit in the multiplier, definitions of mathematical terms in the students' own words, and explanations of errors that occurred on their papers.

Several studies were conducted on teaching college mathematics using writing (Youngberg, 1989; Pallman, 1983; Lesnak, 1989). In one instance, the students were enrolled in four sections of an elementary algebra course, where classes engaged in writing for the last ten minutes of class each day about that day's or the previous day's topic except when a test was scheduled. After the first assignment, the students directed their writing to their classmates and the classmates provided a response after which the instructor responded to both the original writing and the response. The investigator measured student achievement on each of five tests during the semester. On the final exam, the writing group mean was significantly different than the control group, with the writing class

outperforming the control group. Also the writing assignments had a positive impact on achievement for those concepts that were directly related to the writing assignments. Unlike in the previous study (Evans, 1984), where the students with the lowest pretest scores had the greatest gain, in this study, the positive effect was greater for the better students (Youngberg, 1989). Regarding attitudes, the students were neutral about the writing experience with some believing it benefited them and other expressing dislike for the activity. The classes were quite small with 56 total students in the study.

Two investigators have used written explanations of the steps in solving different kinds of problems in teaching remedial college mathematics. In a freshman course, Fundamentals of Mathematics, a mathematics teacher paired with a composition teacher in having the treatment students write detailed explanations of the processes followed in solving different kinds of arithmetic and elementary algebra problems (Pallmann, 1983). The composition teacher included assignments asking the students to explore their feelings about learning mathematics, the relevance of mathematics to their lives, and to pose word problems based on their knowledge of mathematics. The difference in achievement was not significantly different for the experimental and control groups but the retention rate for completing the course was significantly different.: 87% of the treatment group students completed the remedial course while only 39% of the control group was still in the course at completion. The results were considered favorable for the treatment since most of the weaker treatment students were retained in the course while most of the weaker control students dropped the course.

Another study involving writing solution steps in remedial classes was completed in basic algebra classes at the college level. The experiment was designed to test the effect on achievement of using writing to learn activities with these classes (Lesnak, 1989). Using two control and two treatment classes, the investigator found that the writing classes performed significantly better than the nonwriting classes. The student writing consisted mainly of carefully writing the steps to complete in order to solve each type of problem that the students were studying and writing explanations of corrections to problems solved incorrectly. The writing students finished the course with a significantly higher average than the control students (Lesnak, 1989). In addition to the quantitative results, changes in attitude were also reported. These changes were not measured quantitatively but the investigator felt that they were probably the more important result of his study (Lesnak, 1989). Early in the course, the hostility of the writing students bordered on rebellion and by the end, all of the 52 writing students assessed the value of the writing positively, including eight students who did not pass the course. The students reported that the writing helped them to prepare for tests, to identify the material they did not understand, and increased their confidence in their ability to learn algebra. Lesnak believed that the writing activities helped to provide a bridge between mathematical reasoning which his students believed they could not engage in and verbal reasoning which they felt comfortable using.

From these research results, there was reason to believe that writing to learn had potential for a positive effect on mathematical achievement. A discussion of attitudes towards mathematics is included in the next section with some research results.

Attitudes about mathematics

Attitudes towards mathematics have been considered important because national assessment data indicated that attitude and achievement are positively correlated (Dossey, et al., 1988; Crosswhite, 1972). Studies involving attitudes about mathematics usually encompassed beliefs about self and mathematics.(McLeod, 1992). It is theorized that these attitudes developed in two ways, either from automatization of repeated reactions to a mathematical experience or from transfer of an attitude from one task to a related new task (McLeod, 1992). Attitudes about several attributes of mathematics have been measured by researchers: attitudes toward mathematics content, mathematics characteristics, teaching practices, classroom activities, and mathematics teachers (Kulm, 1980). However, most studies have measured various mathematics characteristics such as enjoyment or anxiety.

Although several methods have been used to measure attitudes, such as self-report scales, observations, performance on a set of tasks, or physiological reactions, the majority of studies used self-report scales because of their ease of use. One of the most heavily studied areas was the relationship between attitude and achievement in mathematics, especially during the 1970s (Kulm, 1980). Neither attitude nor achievement seemed dependent on the other, instead, research showed that they interact with each other, sometimes in unpredictable ways (McLeod, 1992; Kulm, 1980). Generally, the studies on attitude improvement proved inconclusive, with some reports of changes of attitude (in both directions), and other results showing no change due to the treatment (Kulm, 1980).

Important work on student attitudes towards mathematics has been reported by Aiken (1974) in developing and validating two scales: the Enjoyment of

Mathematics Scale and the Value of Mathematics Scale. Additional data on the validity of these scales, their correlation with each other, and their correlation with other factors supporting Aiken's contention that the scales measured different aspects of attitude toward mathematics were offered by Watson in her 1983 study. Some research results concerning attitudes of college algebra and precalculus students are included below.

In a study designed to investigate the effects on achievement and attitudes of different strategies of testing in college algebra, the attitude scores for the different modes (quiz, homework, test, and control) were not significantly different although the students strongly preferred the homework mode of testing (Johnson, 1989). Thus the type of testing used did not appear to influence the attitudes of these students. In another study with more than 1000 precalculus students, no significant differences were found between males and females nor due to the size of the student's graduating class from high school, but a significant difference was found related to the student's background in high school mathematics (Stones et al., 1983). Those students with above average high school preparation in mathematics had a significantly more positive attitude towards mathematics than those with average or below average preparation (Stones et al., 1983). Also, freshmen who enrolled in precalculus courses had significantly more positive attitudes towards mathematics than those sophomores and juniors (Stones et al., 1983). The authors suggested that this finding was explained by better mathematics students with more positive attitudes taking mathematics their first year in college while students having poorer attitudes put the mathematics courses off for a year or two. In the next section, a description of the methods used in the present study will be reported.

Methodology

The Sample

The subjects for the study were the students in four intact classes of College Algebra at a midwestern university in the fall 1993 semester. Some students enrolled in the class to fulfill a requirement of their major, others because the course was a prerequisite to another course in their program, and still others took the course to fulfill a general education requirement. Most students had completed three years of high school mathematics, but some had completed four years prior to this course. Many students took Business Calculus subsequent to this College Algebra course while for others College Algebra was their terminal mathematics course.

In the fall 1993 semester, the investigator taught two of the four classes of College Algebra of approximately 50 students each and another instructor taught the other two. A total of 209 students completed the course and the section sizes were similar (Table 1). Each instructor taught one control and one treatment group.

Table 1. Frequency distribution of students in college algebra classes by treatment and by teacher

	Teacher 1	Teacher 2	Total
Treatment			
Writing	52	52	104
Control			
Non-writing	51	54	105
Total	103	106	209

The class met four times weekly (MTWF). The Math Lab was available about 23 hours a week (all day Tuesday and Thursday, one and a half hours per night Sunday through Thursday) where trained tutors worked with students to answer their questions since class interaction was somewhat limited by the size of the sections. These tutorial labs were open-ended in that students could come at their convenience and stay as long as they chose. Students also had the opportunity to join a Supplemental Instruction (SI) group which met for one hour twice a week throughout the semester. The Supplemental Instruction group was an organized study group led by an experienced tutor and designed to help students study the ideas of the course in a small group setting. The six SI groups were limited to a maximum of ten members each. Due to the extra time commitment involved, there was not a problem with more than sixty students requesting participation in a Supplemental Instruction group.

Instructional Materials

The same textbook was used by all classes: *College Algebra, a Graphing Approach* (2nd ed.) by Demana, Waits, and Clemens (1992). This text differed from more traditional ones in that it explored not only algebraic solutions to various types of problems but also graphical solutions. Use of this text required the students and the instructor to use some type of graphing technology. Each student in all sections had his/her own graphing calculator throughout the semester. The calculators were used as a tool for completing homework, quizzes and tests, and in class for explorations and working along with the instructor. Any brand of graphing calculator was acceptable, the instructors and most tutors used TI -81s. The

instructors used TI-81 Viewscreens in the large lecture hall where these classes met.

Procedures

All of the groups studied a variety of topics throughout the semester, with a central theme being the concept of function. Each class studied functions from multiple perspectives with efforts made to connect the perspectives by the use of class activities and discourse. *The treatment group of each instructor spent ten minutes on two days each week completing a directed writing assignment in class.* Writing took place on Mondays and Wednesdays with the papers returned to the students on Tuesdays and Thursdays in the Math Lab. Two days a week were chosen for writing, rather than three or four days a week because of the large number of papers for one instructor to read, and the importance of responding to the writing. Since quizzes and hour exams were scheduled for Fridays, the writing occurred on Mondays and Wednesdays so that students got the feedback before the Friday assessment activity.

Usually the writing took place around the middle of the fifty minute class. This worked well so that latecomers were not excluded. Also if the writing was placed late in the period, some students would begin packing their book bags to leave and this provided a less attentive atmosphere for the writing. The writing prompt concerned an idea discussed in the previous day's class. At a convenient breaking point near the middle of the period, the instructor would remind the students of the topic discussed the day before. Most days, a comment on the importance of the writing process in sorting out one's thinking was also made to the students. Then the instructor asked the students to put away their notes and

presented each student with a page detailing the day's writing assignment. The writing assignments included questions asking students to explain how to solve a problem, why a procedure works the way it does, or to generalize a rule from some explanation or evidence given in the writing stem. The initial assignments included more explaining of how to solve a problem or asking for an interpretation of a solved problem. Later in the study, some of the writing prompts focused on generalizing a rule or explaining the reasoning one would use to work on the problem.

Both instructors emphasized that writing is another way of learning, and reminded the students that the purpose of the writing was to use it as a thinking tool to examine their understanding. After a few assignments, the instructors sensed that some initial hostility toward the writing decreased. Everyone present participated in the writing although some students appeared to make a better effort than others. The complete set of writing assignments appear in Appendix A.

One example of a writing assignment during the study of absolute value inequalities is shown in Figure 1. Another example of a writing assignment while studying systems of inequalities is shown in Figure 2.

A kitchen timer was set for ten minutes, and students finishing early were asked to remain seated and quiet so as not to disturb those still writing. When the timer buzzed, the instructors asked all to finish in one more minute. After the minute, papers were collected. Occasionally, a student had still not finished and handed the paper in at the end of the period.

The instructors spent the time the students were writing in writing also. They wrote to each other analyzing how each felt their teaching was going on that day's concepts, monitoring the class interest and participation, and thinking about

Examine the graph shown for the absolute value inequality given below.
Explain how you can use the graph to see the solution. Using the graph in this way, determine the solution interval(s) for the inequality and write the solution in either inequality or interval form.

$$|2x + 7| \geq 5$$

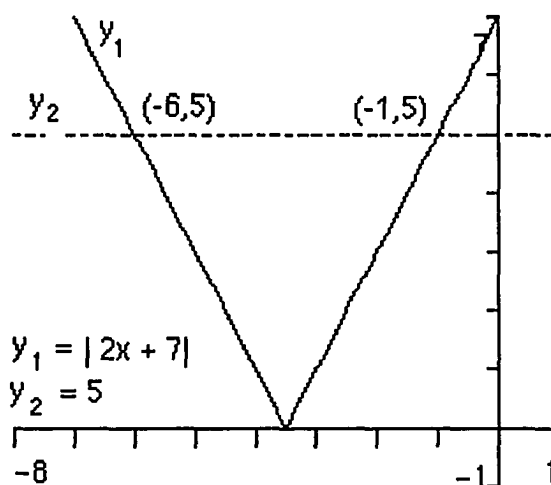


Figure 1. Writing assignment on absolute value inequalities

Imagine that your favorite little brother or sister, a freshman in high school, is coming to visit you this weekend. Further suppose that this sibling notices the following worked problem on a homework sheet and asks you to explain it. Keeping in mind that you're working with a freshman in Algebra 1, explain (1) how you know that one inequality represents a parabola and the other a circle; (2) why one figure is drawn with a dotted "line" and the other with a solid "line"; and (3) how you used the test point (2,0) to determine which region to shade.

$$x^2 + y^2 < 9 \quad \text{Test point (2,0)}$$
$$y \leq -x^2 - 1$$

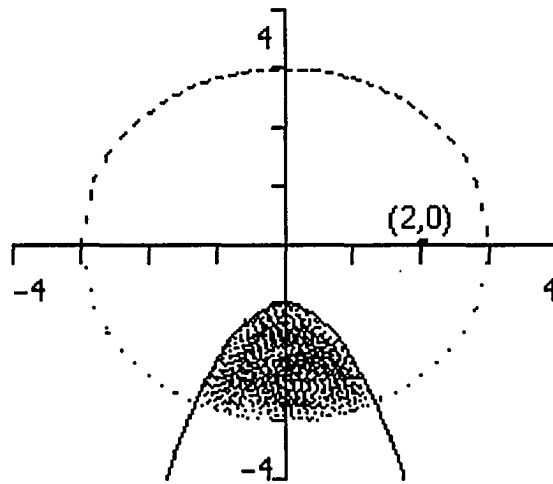


Figure 2. Writing assignment on systems of inequalities

how to improve the presentation or change an example to better communicate a concept. The NCTM's *Professional Standards for Teaching Mathematics* urges mathematics teachers to become more reflective teachers, and this provided an opportunity to begin reflecting on their teaching while engaged in it. The instructors found both the writing and the timer helpful to them as well as to the students: the instructors, by writing, modeled the quiet attention to writing, and they remained seated and did not prepare to resume the class until the buzzer sounded to maintain an attentive atmosphere.

The researcher read all the writing assignments of both treatment classes and provided brief feedback to each student on his/her writing. On papers that had good explanations, the comments were brief: a few words indicating the correctness or complimenting the writer on the good work. An example of such a comment was "yes - good explanation". On papers that were not clear or that contained misunderstandings or errors, the comments were longer, frequently a paragraph or more, explaining the errors or lack of clarity. An example of such a comment on a poor explanation from the papers written in response to the first prompt noted above: "Good start - but the solution is two intervals of x-values, not a region of the coordinate plane. $-1 \leq x \leq -6$ is not true because $-1 \leq -6$. Look for where y_1 lies on or above y_2 . You should be able to 'see' the solution by looking at that graph without using any algebra. Try to learn both solution methods." The researcher alternated the treatment class whose papers were read first each time. Since feedback comments tended to get more succinct after reading many papers, this tactic was used so that one treatment class would not receive more feedback than the other treatment class. Whenever possible, the researcher made a positive or encouraging comment. Attention did not focus on grammar or sentence

structure in the writing but on the mathematics and the understanding displayed. Early in the study, many students had difficulty focusing the information that they knew about a particular topic on the specific question to be addressed. In general, this problem decreased with continued writing.

The writing assignments were not graded. Points were assigned to graded homework assignments, quizzes, and exams but no points in the course grade were attributed to the writing assignments. However, both instructors emphasized to the writing sections the importance of the writing to clarify the student's thinking and to identify concepts that were not understood. Students were encouraged to use the writing for testing their understanding and some students also asked questions within the writing assignments to which the instructor responded. The effects of this clarification would presumably show up in the evaluation measures of quizzes and tests. All other aspects of the treatment and control groups were the same: a common set of lecture notes was used, the same homework assignments were made, and tests and quizzes were similar although not identical.

Instruments

Each student completed a function concept instrument as a pretest and as a posttest to examine the difference in achievement over the semester. The posttest was a part of the final course examination so the students had a strong motivation to do their best. The instrument, adapted by Beverly Rich, is based on an earlier test developed by Greg Foley and used with permission. The reliability coefficient for this instrument was 0.76. The function concept test is in Appendix B.

Also an attitude survey was administered to measure the student's attitudes towards mathematics, their enjoyment and value of mathematics, the importance of

mathematics, and his/her perceived need to know mathematics. The attitude scale was administered at the beginning and at the end of the study to measure changes in attitudes of the groups over the semester. Permission was received to use the released items from the Mathematics Attitude portion of the 1986 National Assessment of Educational Progress (NAEP) as well as Lewis Aiken's Enjoyment of Mathematics and Value of Mathematics Scales (1974). The items numbered 1 through 31 are items from the NAEP, items 32 through 42 are Aiken's Enjoyment Scale and 43 through 52 are Aiken's Value Scale. The reliability coefficient for the entire scale was 0.91. The complete Mathematics Attitude scale is in Appendix C.

Listed below are the eleven items in Aiken's Enjoyment of Mathematics scale, which was revalidated by Watson in 1983. In Aiken's sample of a heterogeneous freshman class, the reliability coefficient was found to be 0.95 (Aiken, 1974), while in Watson's homogeneous sample of students enrolled in a mathematics class the reliability coefficient was 0.88. The reliability coefficient for the students involved in this study was .89.

32. I enjoy going beyond the assigned work and trying to solve new problems in mathematics.
33. Mathematics is enjoyable and stimulating to me.
34. Mathematics makes me feel uneasy and confused.
35. I am interested and willing to use mathematics outside of school and on the job.
36. I have never liked mathematics, and it is my most dreaded subject.
37. I have always enjoyed studying mathematics in school.
38. I would like to develop my mathematical skills.

- 39. Mathematics makes me feel uncomfortable and nervous.
- 40. I am interested and willing to acquire further knowledge of mathematics.
- 41. Mathematics is dull and boring because it leaves no room for personal opinion.
- 42. Mathematics is very interesting, and I have usually enjoyed courses in this subject.

Listed below are the ten items in Aiken's Value of Mathematics scale for which he reported a reliability coefficient of 0.85 (Aiken, 1974). In her 1983 study, Watson reported the reliability on the Value scale as 0.68. The reliability coefficient for the students involved in this study was .79.

- 43. Mathematics has contributed greatly to science and other fields of knowledge.
- 44. Mathematics is less important to people than art or literature.
- 45. Mathematics is not important for the advance of civilization and society.
- 46. Mathematics is a very worthwhile and necessary subject.
- 47. An understanding of mathematics is needed by artists and writers as well as scientists.
- 48. Mathematics helps develop a person's mind and teaches one to think.
- 49. Mathematics is not important in everyday life.
- 50. Mathematics is needed in designing practically everything.
- 51. Mathematics is needed in order to keep the world running.
- 52. There is nothing creative about mathematics; it's just memorizing formulas and things.

Also, students attitudes towards mathematics are generally positively correlated with mathematics achievement. That is, students who are not very

successful at mathematics don't generally have very positive attitudes toward it . Having an attitude measurement as well as an achievement measure would allow the researcher to investigate if the writing treatment was differentially effective on student attitudes.

Demographic data were collected on an instrument developed by the researcher to determine the mathematics background of the subjects including number of math classes completed, length of time since the last math class, and previous use of a calculator. Also, ACT scores as evidence of past achievement were obtained from the university with the permission of the subjects.

A Mathematics Study Process instrument was developed by the researcher to investigate the amount and type of studying, and the attendance practices of the students. The instrument was administered three times during the semester (early, middle, and late in the term). Items on class attendance practices, amount of time spent studying, attendance at the tutorial Math Lab, and specific study actions (reviewing notes, doing homework problems, reviewing errors on previous assignments, etc.) were included. The students were asked to keep a log of their study and Math Lab attendance times for a week each time prior to the collection of these data. For the purposes of this study, the average response of each student on one of the questions was used in order to group students into those who study a great deal, a moderate amount, and very little to examine any differences between the students in the treatment and control sections. The Mathematics Study Process Instrument including the demographic questions appears in Appendix D.

To collect these data on the mathematics study process, the alternate instructor directed the distribution and collection of the surveys in each class. Each

instructor also assured the students that their answers would not impact their grade and that their own instructor would not see their answers.

In each of the four classes, data were collected on the same day. The achievement pretest was administered on September 27 and 28, 1993. The achievement posttest was part of the final exam for all sections which was scheduled on December 21, 1993. The attitude pretest was administered on September 24 and the posttest on December 10, 1993. The three study process questionnaires were completed by students on September 29, October 27, and December 6. This research was approved by both the Iowa State University Human Subjects Review Committee and the Drake University Human Subjects Review Committee.

Data Analysis

The dependent variables for this study were the posttest scores on the function concept test and on the attitude instrument. The independent variables are treatment (control or experimental), ACT scores, time since the last math course, and study habits. The covariate is the pretest function score when analyzing the effect of the other variables on the posttest function score and the covariate is the pretest attitude when analyzing the effect of the treatment on the posttest attitude. The items in the attitude instrument were recoded so that the highest score corresponded to the most positive attitude toward mathematics. The use of a covariate is a method of statistical control of an extraneous variable, in this case, either initial attitude or initial achievement. The covariate partitions out the variance attributed to this variable and to be effective must be linearly related to the dependent variable and unaffected by manipulation of the independent variable.

Neither previous attitude nor previous mathematics achievement would be affected by the writing treatment.

Results

Attitude Results

The data were analyzed using statistical procedures to answer the following questions. The first question investigated by the researcher concerned students' attitudes. **Does the use of in-class writing with feedback improve students' attitudes towards mathematics?**

An analysis of covariance was used to test this hypothesis. The means and standard deviations for the attitude instrument pretest and posttest scores of the four classes are shown in Table 2.

The analysis of variance of posttest attitude scores by treatment and teacher with pretest attitude scores as covariate did not show that the means of the treatment (writing) groups were significantly different than the means of the control (nonwriting) groups (Table 3) by treatment. Thus the researcher's hypothesis that the treatment would improve attitude as measured by this attitude instrument was not supported by these data. Also the analysis did not show any significant effects due to the teacher or to teacher x treatment interaction.

Also, the researcher examined whether the writing treatment might be differentially effective for some students' attitudes more than others. The top third of the students by pretest achievement score were compared to the bottom third of the students by pretest achievement score. Specifically, both treatment and achievement pretest scores were used as independent variables with the attitude

Table 2. Means and standard deviations: attitude pretest and attitude posttest scores for each class by treatment and by teacher

Treatment Teacher	Writing 1	Nonwriting 1	Writing 2	Nonwriting 2	Total
Pretest					
N	52	51	52	54	209
Mean	185.46	184.82	185.81	188.13	186.08
StdDev	19.05	20.37	19.65	22.89	20.46
Possible Range	52 - 260	52 - 260	52 - 260	52 - 260	52 - 260
Posttest					
N	52	51	52	54	209
Mean	188.85	182.55	186.63	189.15	186.84
StdDev	18.24	19.64	22.88	22.27	20.88
Possible Range	52 - 260	52 - 260	52 - 260	52 - 260	52 - 260

Table 3. Analysis of variance of posttest attitude by treatment and by teacher with pretest attitude as covariate

Source of Variation	Sum of Squares	df	Mean Square	F	Sig. of F
Covariate					
Attpre	52811.09	1	52811.09	291.43	.00
Main Effects					
treatment	326.80	1	326.80	1.80	.18
teacher	32.10	1	32.10	.18	.67
Interaction					
Residual	36968.12	204	181.22	3.05	.08

posttest score as dependent variable and attitude pretest as covariate. The means and standard deviations for this analysis are given in Table 4.

The analysis of variance of posttest attitude scores by treatment and by pretest achievement scores on the top and bottom thirds of each group with pretest attitude scores as covariate showed that the means of the treatment (writing) groups were significantly different than the means of the control (nonwriting) groups (Table 5) by treatment. The mean of the low achievement writing students was higher on the attitude posttest than the mean of the low achievement nonwriting students. But the same was not true of the high achievement students. The nonwriting high achievement students had a more positive attitude posttest mean than the high achievement writing students. Thus, for this sample, the writing treatment was more effective on attitude for the low achievement students. Also the analysis did not show any significant effects due to the achievement pretest score or to teacher x achievement interaction.

Table 4. Means and standard deviations: attitude posttest scores by treatment and by achievement pretest scores for the top and bottom thirds

Treatment	Writing	Nonwriting	Writing	Nonwriting
Achievement	Low	Low	High	High
Posttest				
N	33	43	39	34
Mean	186.03	180.35	189.64	194.32
StdDev	20.63	20.47	18.58	17.74
Possible				
Range	52 - 260	52 - 260	52 - 260	52 - 260

Table 5. Analysis of variance of posttest attitude score by previous achievement level and treatment with pretest attitude score as covariate

Source of Variation	Sum of Squares	df	Mean Square	F	Sig. of F
Covariate					
Attpre	38713.74	1	38713.74	291.42	.00
Main Effects	857.95	2	428.98	3.23	.04
achievement	57.91	1	57.91	.43	.51
treatment	734.60	1	734.60	5.53	.02
Interaction	43.12	1	43.12	.33	.57
Residual	19129.43	144	132.84		

Next, the researcher examined the differences in attitude posttest scores on the Aiken Enjoyment of Mathematics scale. An analysis of variance of posttest Aiken Enjoyment of Mathematics scores by treatment and teacher with Aiken Enjoyment of Mathematics scale pretest scores as covariate was performed. The means and standard deviations for the Aiken Enjoyment of Mathematics scale pretest and posttest scores of the four classes are shown in Table 6. The analysis of variance of posttest Aiken Enjoyment of Mathematics scores by treatment and teacher with Aiken Enjoyment of Mathematics scale pretest scores as covariate did not show a significant difference due to treatment as shown in Table 7. Also, the analysis did not show a significant difference due to teacher or to treatment x teacher interaction.

Also, the researcher examined the differences in attitude posttest scores on the Aiken Value of Mathematics scale. An analysis of variance of posttest Aiken

Table 6. Means and standard deviations: Aiken Enjoyment scale pretest and Aiken Enjoyment scale posttest scores for each class

Treatment	Writing	Nonwriting	Writing	Nonwriting	Total
Teacher	1	1	2	2	
Pretest					
N	52	51	52	54	209
Mean	33.50	34.14	33.48	34.87	34.00
StdDev	7.73	7.91	8.23	8.98	8.20
Possible					
Range	11 - 55	11 - 55	11 - 55	11 - 55	11 - 55
Posttest					
N	52	51	52	54	209
Mean	34.29	32.90	34.35	34.83	34.11
StdDev	7.51	8.14	8.68	8.66	8.24
Possible					
Range	11 - 55	11 - 55	11 - 55	11 - 55	11 - 55

Table 7. Analysis of variance of Aiken Enjoyment posttest score by treatment and teacher with Aiken Enjoyment pretest score as covariate

Source of Variation	Sum of Squares	df	Mean Square	F	Sig. of F
Covariate					
Achpre	8943.52	1	8943.52	361.43	.00
Main Effects					
treatment	107.16	2	53.58	2.17	.12
teacher	81.99	1	81.99	3.31	.07
	26.34	1	26.34	1.07	.30
Interaction					
	21.03	1	21.03	.85	.36
Residual	5047.98	204	24.75		

Value of Mathematics scores by treatment and teacher with Aiken Value of Mathematics scale pretest scores as covariate was performed. The means and standard deviations for the Aiken Value of Mathematics scale pretest and posttest scores of the four classes are shown in Table 8.

The analysis of variance of posttest Aiken Value of Mathematics scores by treatment and teacher with Aiken Value of Mathematics scale pretest scores as covariate did not show a significant difference due to treatment as shown in Table 9. Also, the analysis did not show a significant difference due to teacher but did show a significant difference due to treatment x teacher interaction. The writing

Table 8. Means and standard deviations: Aiken Value scale pretest and Aiken Value scale posttest scores for each class

Treatment	Writing	Nonwriting	Writing	Nonwriting	Total
Teacher	1	1	2	2	
Pretest					
N	52	51	52	54	209
Mean	38.35	39.06	39.15	39.30	38.97
StdDev	5.71	4.49	5.12	4.85	5.04
Possible					
Range	10 - 50	10 - 50	10 - 50	10 - 50	10 - 50
Posttest					
N	52	51	52	54	209
Mean	40.00	38.59	39.00	39.80	39.35
StdDev	4.87	5.26	6.36	4.94	5.38
Possible					
Range	10 - 50	10 - 50	10 - 50	10 - 50	10 - 50

Table 9. Analysis of variance of Aiken Value posttest score by treatment and teacher with Aiken Value pretest score as covariate

Source of Variation	Sum of Squares	df	Mean Square	F	Sig. of F
Covariate					
Achpre	1828.88	1	1828.88	91.38	.00
Main Effects	17.56	2	8.78	.44	.65
treatment	15.35	1	15.35	.77	.38
teacher	2.07	1	2.07	.10	.75
Interaction	84.66	1	84.66	4.23	.04
Residual	4082.70	204	20.01		

students of teacher 1 showed more positive attitudes on the Aiken Value posttest than the nonwriting students of teacher 1. Just the opposite was the case for teacher 2. The nonwriting students of teacher 2 had the more positive attitudes on the Aiken Value posttest than the writing students of teacher 2 as shown Figure 3.

A factor analysis of the entire attitude instrument was conducted to identify factors in the scale. The results were quite similar to the two Aiken factors of enjoyment and value. The attitude items identified as belonging to the two factors as well as the statistical analysis of them are contained in Appendix E.

Achievement Results

The second question investigated by the researcher concerned achievement: **Does the use of in-class writing with feedback improve**

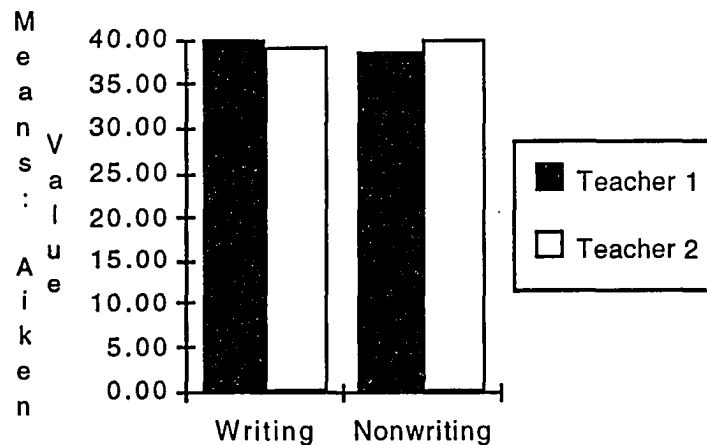


Figure 3. Interaction of treatment and teacher

students' performance on a general concept test dealing with functions?

An analysis of covariance was used to test this hypothesis. The means and standard deviations for the achievement pretest and posttest scores of the four classes are shown in Table 10.

The analysis of variance of posttest achievement scores by treatment and teacher with pretest achievement scores as covariate did not show that the means of the treatment (writing) groups were significantly different than the means of the control (nonwriting) group as shown in Table 11. Thus the researcher's hypothesis that the treatment would improve achievement as measured by this instrument was not supported by these data. Also the analysis did not show any significant effects due to the teacher or to teacher x treatment interaction.

Table 10. Means and standard deviations: achievement pretest and achievement posttest scores for each class

Treatment Teacher	Writing 1	Nonwriting 1	Writing 2	Nonwriting 2	Total
Pretest					
N	52	51	52	54	209
Mean	15.04	12.78	13.65	14.44	13.99
StdDev	5.46	5.36	4.59	5.05	5.16
Possible Range	0 - 35	0 - 35	0 - 35	0 - 35	0 - 35
Posttest					
N	52	51	52	54	209
Mean	21.83	20.59	22.08	22.26	21.70
StdDev	5.39	5.44	3.74	4.75	4.88
Possible Range	0 - 35	0 - 35	0 - 35	0 - 35	0 - 35

Table 11. Analysis of variance of posttest achievement by treatment and by teacher with pretest achievement as covariate

Source of Variation	Sum of Squares	df	Mean Square	F	Sig. of F
Covariate					
Achpre	2544.85	1	2544.85	218.82	.00
Main Effects					
treatment	.07	1	.07	.01	.94
teacher	39.147	1	39.15	3.37	.07
Interaction					
Residual	5.47	1	5.47	.47	.49
	2372.51	204	11.63		

The third question under investigation concerned treatment effect on students with different levels of previous mathematics achievement: **Is the in-class writing with feedback treatment differentially effective for students with varying levels of mathematics achievement as measured by ACT scores?**

The university does not require ACT scores for transfer or foreign students so this data is missing for 21 students in the study (10%). For the students for whom ACT scores were available, the researcher compared the means of the lower third of students by ACT composite score (21 or less) with those in the upper third of ACT composite scores (26 or more). An analysis of variance of posttest achievement scores by treatment and ACT composite scores with pretest achievement as covariate was performed. The means and standard deviations for these pretest and posttest achievement scores by treatment and by ACT scores is shown in Table 12.

The analysis of variance of posttest achievement scores by treatment and ACT composite scores with pretest achievement scores as covariate did not show that the means of the treatment (writing) groups were significantly different than the means of the control (nonwriting) groups (Table 13) by treatment. However the analysis did show that the means of the groups were significantly different by ACT scores with students with higher ACT composite scores having a higher posttest average than those with lower ACT composites ($n = 104$). This was expected since previous achievement bears on mathematics learning. No significant interaction was found for treatment x ACT scores.

The fourth question investigated by the researcher involved the effect of the writing treatment on students whose time since their last mathematics class varied:

Table 12. Means and standard deviations: achievement pretest and achievement posttest scores for each class by ACT scores

Treatment ACT score	Writing Low	Writing High	Nonwriting Low	Nonwriting High	Total
Pretest					
N	29	28	20	27	104
Mean	10.97	18.07	10.45	17.04	14.36
StdDev	3.73	4.82	3.49	5.01	5.50
Possible Range	0 - 35	0 - 35	0 - 35	0 - 35	0 - 35
Posttest					
N	29	28	20	27	104
Mean	18.48	25.21	17.70	24.59	21.73
StdDev	3.86	4.09	4.73	3.72	5.26
Possible Range	0 - 35	0 - 35	0 - 35	0 - 35	0 - 35

Table 13. Analysis of variance of posttest achievement by ACT scores and by treatment with pretest achievement as covariate

Source of Variation	Sum of Squares	df	Mean Square	F	Sig. of F
Covariate					
Achpre	1606.80	1	1606.80	146.08	.00
Main Effects					
ACT	147.50	2	73.75	6.71	.00
Treatment	147.48	1	1.48	13.41	.00
Interaction	1.77	1	1.77	.16	.69
Residual	1.25	1	1.25	.11	.74
	1088.91	99	11.00		

Is the in-class writing with feedback treatment differentially effective for students with varying length of time since completion of the previous math class?

Since nearly half (45.9%) of the students in the study had completed their last math class four months ago or less, the posttest achievement mean of this group was compared to the mean of those who had completed their last math class one and a half years ago or more (15.3%) to see if there was a significant difference due to the treatment. An analysis of variance of posttest achievement scores by treatment and length of time since the last math class with pretest achievement scores as covariate was performed. The means and standard deviations of the achievement pretests and posttests by treatment and by time since the last math class are given in Table 14.

The analysis of variance of posttest achievement scores by treatment and length of time since the last math class with pretest achievement scores as covariate did not show that the means of the treatment (writing) groups were significantly different than the means of the control (nonwriting) groups by treatment (Table 15). Nor was there a significant difference in the means of the groups by time since the last math class. There was, however, a significant interaction between the treatment and time since the last math class. The students in the writing group who had not taken a math class recently scored significantly better than those in the control group who had not taken a math class recently as shown in the graph of the interaction in Figure 4. The control group students who had recently completed a math course outscored the writing group students who had recently completed a math course.

Table 14. Means and standard deviations: achievement pretest and achievement posttest scores for each class by time since the last math class

Treatment	Writing	Nonwriting	Writing	Nonwriting	Total
Time	< 4 mo.	< 4 mo.	> 1.5 yrs.	> 1.5 yrs	
Pretest					
N	52	44	26	33	155
Mean	15.19	14.61	12.69	13.58	14.26
StdDev	4.77	5.70	4.57	5.12	5.13
Possible					
Range	0 - 35	0 - 35	0 - 35	0 - 35	0 - 35
Posttest					
N	52	44	26	33	155
Mean	21.79	22.27	22.85	20.82	21.90
StdDev	4.52	4.69	4.17	5.51	4.74
Possible					
Range	0 - 35	0 - 35	0 - 35	0 - 35	0 - 35

Table 15. Analysis of variance of posttest achievement by treatment and by time since the last math class with pretest achievement as covariate

Source of Variation	Sum of Squares	df	Mean Square	F	Sig. of F
Covariate					
Achpre	1661.22	1	1661.22	150.37	.00
Main Effects	32.88	2	16.44	1.49	.23
time since	27.52	1	27.52	2.50	.12
treatment	7.83	1	7.83	.71	.40
Interaction	109.16	1	109.16	9.88	.00
Residual	1657.10	150	11.05		

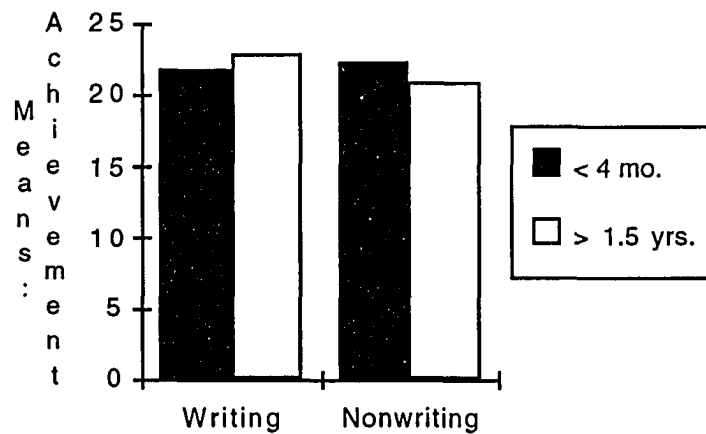


Figure 4. Interaction of treatment and time since the last math class

The final question investigated by the researcher involved the treatment effect on the achievement of students who differed by the amount of time spent studying: **Is the in-class writing with feedback treatment differentially effective for students with varying self-recorded mathematics study habits?**

The researcher computed each student's average study time by computing the mean of the three study times reported on the Study Process Instrument. Then the means of the lower third of students by study time (6 hours or less per week) were compared with those in the upper third of reported study time (more than 8.5 hours per week). The means and standard deviations of the pretest and posttest achievement scores by treatment and by study time are shown in Table 16.

The analysis of variance of posttest achievement score by treatment and average study time with pretest achievement score as covariate did not show that the means of the treatment (writing) groups were significantly different than the means of the control (nonwriting) groups (Table 17) by treatment. A significant difference was not found between the means of the groups by average study time nor was there a significant interaction for study time x treatment (Table 17).

Discussion

Unlike in some other studies (Smith et al., 1992, Lesnak, 1989), the data of this study did not show that overall student attitudes toward mathematics were significantly more positive for the treatment students at the end of the course. It may be that any change that occurred was not on the attitudes measured in the chosen instrument. Or it may be that attitudes are more stable (McLeod, 1992) and

Table 16. Means and standard deviations: achievement pretest and achievement posttest scores for each class by time spent studying

Treatment Studytime	Writing Low	Nonwriting Low	Writing High	Nonwriting High	Total
Pretest					
N	32	31	36	35	134
Mean	16.78	15.84	11.89	11.66	13.91
StdDev	6.20	5.73	3.54	3.56	5.31
Possible Range	0 - 35	0 - 35	0 - 35	0 - 35	0 - 35
Posttest					
N	32	31	36	35	134
Mean	23.88	22.55	20.28	20.14	21.63
StdDev	4.38	5.53	4.87	4.38	5.00
Possible Range	0 - 35	0 - 35	0 - 35	0 - 35	0 - 35

Table 17. Analysis of variance of posttest achievement by treatment and by average study time with pretest achievement as covariate

Source of Variation	Sum of Squares	df	Mean Square	F	Sig. of F
Covariate					
Achpre	1858.84	1	1858.84	164.43	.00
Main Effects					
study time	4.29	2	2.14	.19	.83
treatment	1.33	1	1.33	.12	.73
Interactions	2.86	1	2.86	.25	.62
Residual	3.93	1	3.93	.35	.56
	1458.29	129	11.31		

would change more slowly than over the course of a single three month period. The analysis of variance of posttest attitude scores completed on the top third and the bottom third of the classes by pretest achievement and by treatment showed that the writing treatment was more beneficial to the low achievers in improving their attitude towards mathematics. This may be because the writing assignments empowered them to make connections and explore relationships using a learning technique that was more suitable for them or that they felt more comfortable with (Lesnak, 1989). Writing activities appeared to provide a link between the verbal reasoning skills that the students were confident in using and the mathematical reasoning skills that they were struggling with learning (Lesnak, 1989). This finding, if replicated by others, has practical implications in mathematics classrooms for making mathematics more accessible to all students. This learning strategy may offer opportunities for previously less successful mathematics students to study more mathematics. If one feels more positive about a subject it is more likely that one will be open to learning more in that area, since attitude and achievement are positively correlated (Dossey, 1988). The NCTM *Standards* (1989) call for a richer curriculum in mathematics to be studied by all students and more positive attitudes are one important component in accomplishing this change.

Most all of the students were unaccustomed to writing about mathematics and found the writing to be quite difficult to do. This may have been a confounding factor in this study that interfered with the potential of the treatment. The instructors tried hard to give encouragement to the class as a whole as well as to individual students on their papers when progress became apparent. In anecdotal reports, many students did state that the writing became easier as the semester progressed and that trying to express in words their understanding did help them to identify

concepts that they did not understand. Additional development of activities to help students learn to write about mathematics would be a useful addition for students and teachers.

The analysis of variance of posttest attitudes for the Aiken Enjoyment scale and the Aiken Value scale did not show a significant difference for the treatment groups. This is probably related to the reasons stated above. However, there was a significant interaction for treatment x teacher on the Aiken Value of Mathematics scale. This may not have much practical implication. Teacher 1, by virtue of being the researcher who believed strongly in the writing treatment, may have communicated more of that conviction to her writing students than the other teacher. However, attitude development and change are subtle processes whose interaction is not well understood (McLeod, 1992).

The analysis of variance for the posttest achievement scores by teacher and by treatment did not show a significant effect on achievement for the writing students. In Youngberg's study (1989), which did show a significant difference for achievement for the writing students, she stated that the effect was greatest on questions which directly related to the writing exercises. Youngberg used several unit tests throughout the semester and a final exam to measure differences rather than the single pretest and posttest design of this study. It may be that the writing prompts in this study were not as closely aligned with questions on the posttest as in the Youngberg study. Also, Youngberg's students wrote everyday rather than twice a week so the greater intensity of treatment may also have had an influence on the outcome.

Regarding the differential achievement effect for students with varying levels of previous achievement as measured by ACT scores, the analysis of variance

showed a significant effect for ACT score. This is not a surprising finding nor one with much practical implication in the mathematics classroom since previous achievement bears on mathematical learning.

Often nontraditional students or those who have not taken mathematics courses in the previous semester or year struggle in College Algebra because they find it difficult to keep up with the pace of the course. The adage "use it or lose it" seems to be appropriate for algebraic symbol manipulation skills. The significant interaction of treatment and time since the last mathematics course in favor of the writing students who had not recently completed a mathematics course provides evidence of another way that writing to learn can make mathematics accessible to certain students. This finding, if replicated by others, may have practical implications for mathematics classrooms.

In the analysis of differential effect of study time and treatment on achievement scores, no significant differences were found between the groups for treatment or for their amount of time spent studying. The students who reported high study times actually had lower achievement scores than those who reported less time spent studying. But those reporting higher study hours gained slightly more points on the achievement score posttest than the students who studied less. This may be an indication that the students with less incoming achievement worked longer (harder or perhaps less efficiently) for the learning that they achieved.

Limitations

If other changes occurred for the writing students in this study, they were lost in the variations among students or the measuring instruments may have been too coarse to record them. The majority of the students in this study were entering

freshman who had many adjustments to make to collegiate life. The ease and timing of these adjustments often impact first semester academic performance and may have confounded some of the results of this study.

The fact that the researcher was also one of the teachers involved in the study may have introduced some bias into the results as indicated earlier. Also, the length of time of the study, most of a single semester, may not have been a long enough time for attitude or achievement changes to occur. Also, the similarity of the writing assignments to the achievement items may not have been strong enough. Finally the intensity of the treatment may not have been great enough. Either longer writing activities or more of them may have produced different results. Extensions of these findings should be approached cautiously.

Recommendations

Additional controlled studies will be useful for finding more evidence of how writing to learn affects achievement and attitudes. Using some of the alternate methods of feedback, such as peer review or checksheets for form and content would make writing every day more feasible. It would seem that students need some experience with writing in mathematics before they begin to respond to each other's papers. Their initial writings were rather unfocused and would have provided a difficult challenge for their peers.

Another change for intensifying the treatment might include writing assignments outside of class such as microthemes that are typed, one page only papers. The reading of the writing assignments in the study was slowed considerably by the difficulty of reading students' handwritten papers. A final change that might be examined is giving credit, whether a grade or simply credit for

completion of the writing assignments. Some students may give more care and attention to the writing assignments if they can see more directly the benefit that completion of the writing assignments has on their grade. There is not agreement in the literature about whether points should be assigned for writing but points for completion might be very motivational to students. Using the potential of electronic communications in a subsequent study by receiving the writing assignments via e-mail and sending responses that way might prove to be more time efficient for the instructor. Also if a news group were established for the students in the class, it would be possible to share several examples of good responses that all the students could see. Since usually more than one approach is possible, this would be a way of validating multiple correct responses.

GENERAL CONCLUSIONS

The literature review in the first article showed both research results and practitioner support of writing to learn mathematics. The study results in the second article give some support to the writing treatment as beneficial for certain students, although overall significant gains in attitude or achievement were not found. The writing treatment group of low achievers showed a significant difference in positive attitudes towards mathematics in the posttest attitude measurement. The most important implications of a study such as this one relate to curriculum and course improvement. The use of in-class writing to learn mathematics is one option for implementing the NCTM *Standards* call for students to be able to communicate their mathematical ideas in writing. The use of writing also increases the communication between the instructor and the student, and provides an alternate form of communication for students who may not participate actively in class. Use of writing can be a barometer for the instructor in planning or revising the curriculum. These uses of writing are fruitful areas for additional studies.

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APPENDIX A WRITING ASSIGNMENTS

Writing #1
Oct. 4, 1993 Section_____

97
Name _____
SSN _____

Examine the algebraic and graphical solutions below to the given inequality problem. Then explain to a classmate who was absent how both of the graphs show the same solution to the inequality as the algebraic method. Write as clearly as you can.

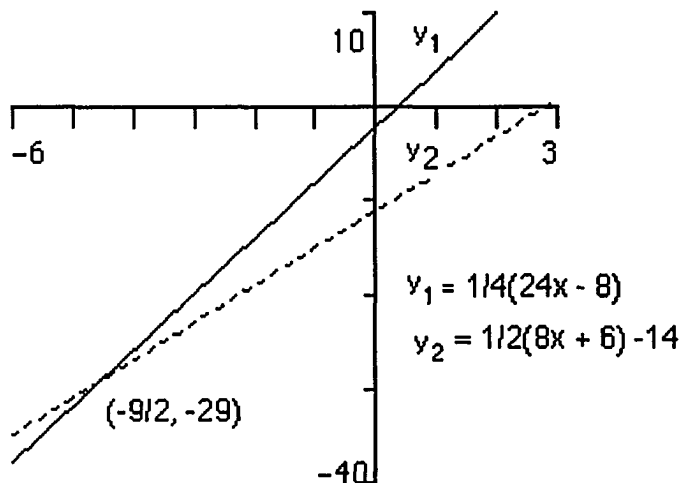
$$\frac{1}{4}(24x - 8) < \frac{1}{2}(8x + 6) - 14$$

$$6x - 2 < 4x + 3 - 14$$

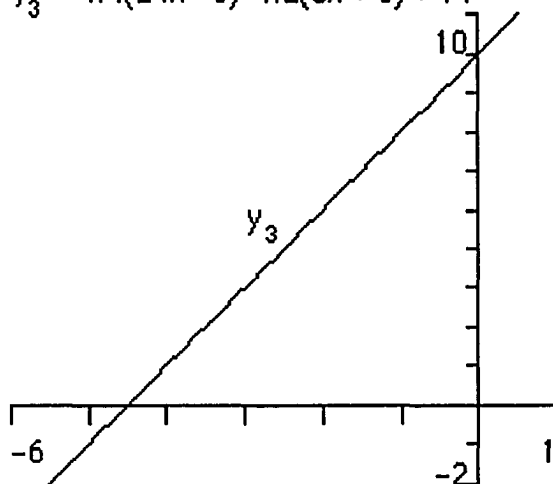
$$6x - 2 < 4x - 11$$

$$2x < -9$$

$$x < \frac{-9}{2}$$



$$y_3 = \frac{1}{4}(24x - 8) - \frac{1}{2}(8x + 6) + 14$$



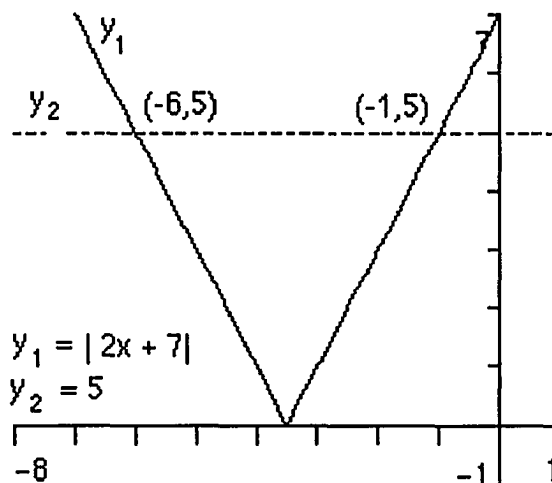
Writing #2
Oct. 6, 1993

Section_____

Name_____
SSN_____

Examine the graph shown for the absolute value inequality given below.
Explain how you can use the graph to see the solution. Using the graph
in this way, determine the solution interval(s) for the inequality and
write the solution in either inequality or interval form.

$$|2x + 7| \geq 5$$



Writing #3

Oct. 11, 1993

Section_____

Name_____

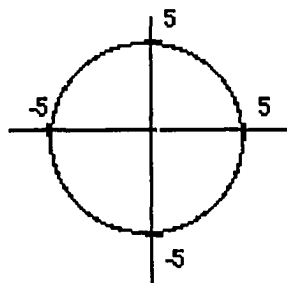
SSN_____

Examine the equation and the graph of the relation given below.

(1) Then explain to a classmate who was absent what must be true about (a, b) if (a, b) is a solution of the relation. In other words, how are "a" and "b" related?

(2) What other names have we given to the term "solution" this semester? Do these other names fit this example? Write as clearly as you can.

$$x^2 + y^2 = 25$$



Writing #4

Oct. 13, 1993

Section_____

Name_____

SSN_____

Explain the difference between testing for symmetry with respect to the y-axis and symmetry with respect to the x-axis. Be sure to discuss both the algebraic and geometric tests and write as clearly as you can.

Writing #5

Oct. 18, 1993

Section_____

Name_____

SSN_____

Function: A function of x is a relation with the following property: If both (x, y_1) and (x, y_2) belong to the relation, then $y_1 = y_2$.

Vertical line test for function: If every vertical line intersects the graph of a relation in at most one point, then the relation is a function of x .

Explain how these two definitions for a function are related. If it helps you to think more clearly, use examples such as $y = x^2$ and $x = y^2$ to explain the relationship between these definitions.

Writing #6
Oct. 20, 1993

Section_____

Name_____

SSN_____

Tickets to a concert on campus cost \$5.00 for students and \$7.75 for nonstudents. Suppose x represents the number of student tickets sold and y represents the number of nonstudent tickets sold. The inequality that describes the condition that total receipts must exceed \$3000 is:

$$5x + 7.75y > 3000$$

Suppose you want a complete graph of this problem situation. Discuss whether your graph should include all points above a certain line or only a square grid of points above a certain line. Defend your decision.

Writing #7

Oct. 25, 1993

Section_____

Name_____

SSN_____

In the quadratic equation $y = a(x - h)^2 + k$, we have discussed the effect that a , h , and k have on the benchmark parabola.

Explain to a classmate who was absent the effect that both the size and the sign of " a " have on the benchmark parabola. Write this as clearly as you can. The better you can explain this, the better you yourself understand it.

Writing #8

Oct. 27, 1993

Section_____

Name_____

SSN_____

What effect does the order of geometric transformations have on the benchmark parabola, $y = x^2$? Specifically, does a vertical stretch followed by a vertical shift have the same effect as a vertical shift followed by a vertical stretch? Discuss this either in terms "a" and "k" in the general parabola equation, or use a specific numerical example if that is easier for you to write about. Explain as clearly as you can.

Writing #9
Nov. 1, 1993

Section_____

Name_____

SSN_____

For the functions $f(x)$ and $g(x)$ described below, discuss the domain of each function. Explain why the domain of their sum (D_{f+g}) is the intervals given. Explain the significance of the parentheses and brackets in the domain of the sum. Write as clearly as you can.

$$f(x) = \sqrt{2 - x}$$

$$g(x) = \frac{1}{x + 2}$$

$$(f + g)(x) = \sqrt{2 - x} + \frac{1}{x + 2}$$

$$D_{f+g}: (-\infty, -2) \cup (-2, 2]$$

Writing #10

Nov. 3, 1993

Section_____

Name_____

SSN_____

What is the relationship between the graph of $f(x)$ and $f^{-1}(x)$? What does the graph of $y = x$ have to do with the graph of a function and its inverse? What can you say about the ordered pairs that belong to $f(x)$ and $f^{-1}(x)$?

Do this in general if you can but if you need a specific example to help you think, use $f(x) = 3x + 2$ and its inverse to discuss the three questions above.

Writing #11

Nov. 8, 1993

Section_____

Name_____

SSN_____

In class we discussed and solved systems of linear equations. For two equations in two variables, explain the three cases possible for the solution of a system of two lines. Also explain how you will recognize each of the three cases from the results of the algebra you do. Write as clearly as you can.

Writing #12
Nov. 10, 1993

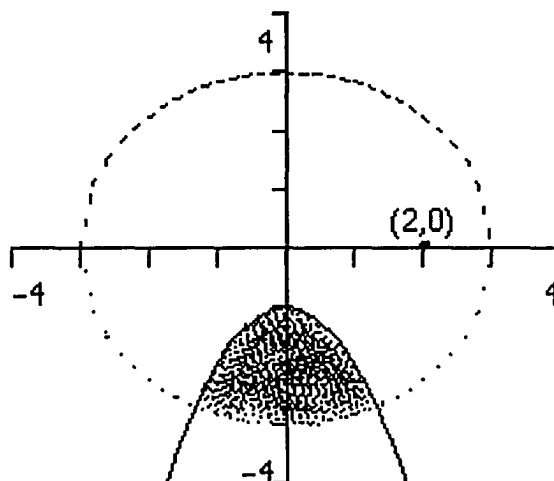
Section_____

Name_____
SSN_____

Imagine that your favorite little brother or sister, a freshman in high school, is coming to visit you this weekend. Further suppose that this sibling notices the following worked problem on a homework sheet and asks you to explain it. Keeping in mind that you're working with a freshman in Algebra 1, explain (1) how you know that one inequality represents a parabola and the other a circle; (2) why one figure is drawn with a dotted "line" and the other with a solid "line"; and (3) how you used the test point (2,0) to determine which region to shade.

$$x^2 + y^2 < 9$$
$$y \leq -x^2 - 1$$

Test point (2,0)



Writing #13

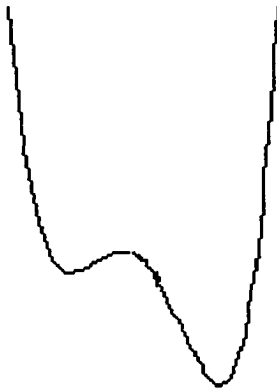
Nov. 15, 1993

Section_____

Name_____

SSN_____

A friend, Chris, received an assignment to sketch the shape of the graph of a function. Unfortunately, the ink was smeared in the rain and she could only read the first term of the polynomial, which was $4x^7$. Chris drew the sketch shown below. Write a letter to Chris agreeing or disagreeing with her sketch of the shape of the curve. Be as specific as you can about your reasons for agreeing or disagreeing.



Writing #14
Nov. 17, 1993

Section_____

Name_____

SSN_____

In Section 4.1, you completed work on the following problem situation: The total daily revenue of a lemonade stand at a state fair is given by the equation $R = xp$, where x is the number of glasses of lemonade sold daily and p is the price of one glass of lemonade. Assume that the price of lemonade is given by the "supply" equation $p = 2 + 0.002x - 0.0001x^2$. In your homework, you found the revenue function, a complete graph of this function and the number of glasses of lemonade to be sold to produce maximum daily revenue. (#54 - 56).

Conventional wisdom says that management should work to continually increase sales. Write a paragraph that either supports or refutes this point of view in the case of this problem situation. Be as specific as you can about the reasons for your position.

Writing #15

Nov. 22, 1993

Section_____

Name_____

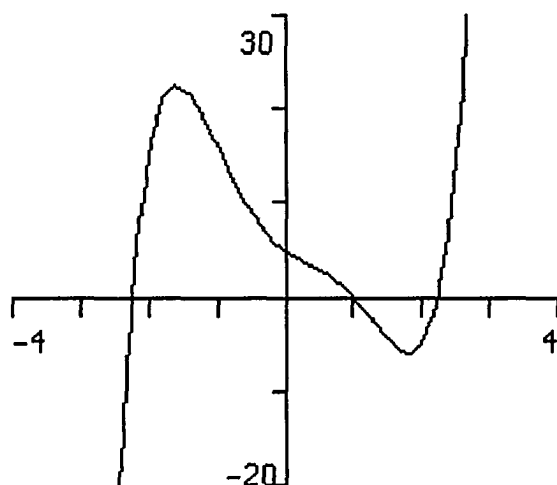
SSN_____

Chris was studying early for the next exam and concluded that if a polynomial had any real zeros then they could be found using the rational zeros theorem. Any other zeros would be complex imaginary and could be found by reducing the order of the polynomial and using the quadratic formula.

However, then Chris tried to solve the following problem:

$$f(x) = x^5 - x^4 - 4x^3 + 4x^2 - 5x + 5 = 0.$$

From the rational zeros theorem, Chris knew that the only rational zeros possible are: +1, -1, +5, -5. Using the calculator, Chris saw the graph below. Explain to Chris the error in the reasoning of the first paragraph.



Writing #16

Nov. 29, 1993

Section_____

Name_____

SSN_____

In the rational equation, $y = \frac{a}{x - h} + k$, we have discussed the effect that "a", "h", and "k" have on the benchmark rational function, $f(x) = \frac{1}{x}$.

Explain to a classmate who was absent the effect that "h", "k" and both the size and the sign of "a" have on the benchmark rational function. Write this as clearly as you can. The better you can explain this, the better you yourself understand it.

Writing #17

Dec. 1, 1993

Section_____

Name_____

SSN_____

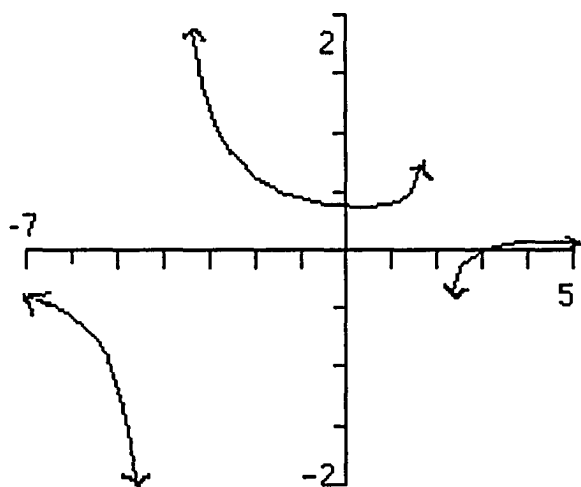
For the rational function given below, discuss

(a) how to find the vertical asymptote(s) and the horizontal asymptote and why one finds them (what's important about them other than helping you draw the graph?),

(b) how to find the x-intercept(s) and the y-intercept.

Since the graph is already drawn and labeled you needn't actually do any algebra to find these, but rather discuss in general how you find these values for a rational function.

$$f(x) = \frac{x - 3}{x^2 + 2x - 8}$$



APPENDIX B FUNCTION CONCEPT TEST

Test Directions

You are permitted to use a calculator for these items. You may write on the test booklet. PLEASE ENTER YOUR SOCIAL SECURITY NUMBER ON THE ANSWER SHEET AND BELOW ON THE TEST BOOKLET.

In each item, you will be asked to answer a question, complete a statement or solve an equation or inequality. Once you have decided upon an answer to a problem, PLEASE FILL IN THE LETTER OF YOUR ANSWER WITH A #2 PENCIL ON THE ANSWER SHEET.

Please try to answer all items. There is no penalty for guessing. If you happen to get stuck on a particular item, move on and come back to it later. You may do any scratch work on the test booklet itself.

Social Security Number _ _ _ _ - _ _ _ - _ _ _ _

1. A function f with domain $\{1, 2, 3\}$ is defined by $f(x) = 2^x$. The range of f is

A. $\{\frac{1}{4}, \frac{1}{2}, \frac{1}{8}\}$
B. $\{\frac{1}{2}, 1, 1\frac{1}{2}\}$
C. $\{1, 2, 4\}$
D. $\{2, 4, 6\}$
E. $\{2, 4, 8\}$

2. The equation of line l is $y = 4x - 5$

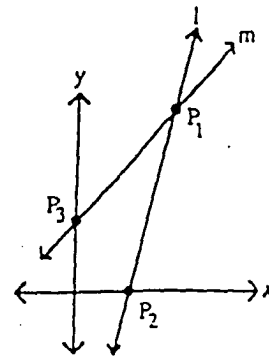
The equation of line m is $y = 2x + 2$

What is the solution of the simultaneous equations

$$y = 4x - 5$$

$$y = 2x + 2$$

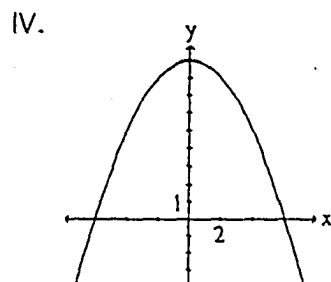
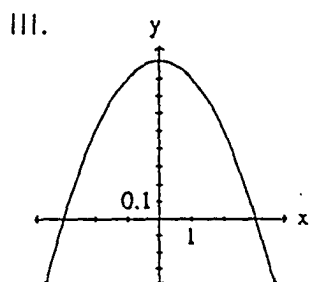
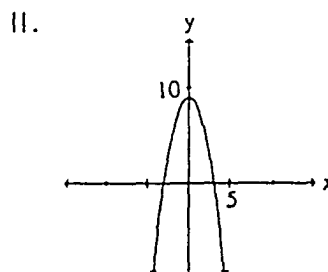
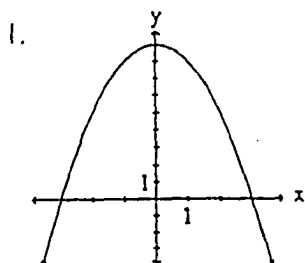
- A. the coordinates of P_1
B. the coordinates of P_2
C. the coordinates of P_3
D. the x -value at P_2 and the y -value at P_3
E. the y -value at P_2 and the x -value at P_3



3. The complex number $(1 + i)^2$ is equal to

A. 0
B. 2
C. $2i$
D. $1 + i$
E. $2 + 2i$

4. Which are graphs of the same set of ordered pairs?



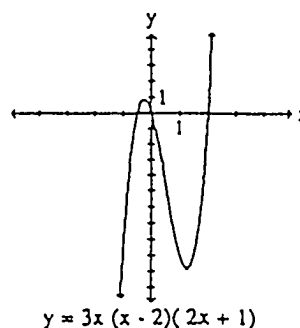
- A. I and II
 - B. II and III
 - C. III and IV
 - D. I and IV
 - E. II and IV
5. If $xy = 1$ and x is greater than 0, which of the following statements is true?
- A. When x is greater than 1, y is negative.
 - B. When x is greater than 1, y is greater than 1.
 - C. When x is less than 1, y is less than 1.
 - D. As x increases, y increases.
 - E. As x increases, y decreases.

6. The graph of the curve $y = \frac{2x + 1}{x^2 + 2x + 3}$ intersects the axes at the points

- A. $(\frac{1}{2}, 0)$ and $(0, -\frac{1}{3})$
- B. $(-\frac{1}{3}, 0)$ and $(0, \frac{1}{2})$
- C. $(\frac{1}{2}, 0)$ and $(0, \frac{1}{3})$
- D. $(-\frac{1}{2}, \frac{1}{3})$ and $(0, 0)$
- E. $(-\frac{1}{2}, 0)$ and $(0, \frac{1}{3})$

7. The curve defined by $y = 3x(x - 2)(2x + 1)$ intersects the x-axis only at the points

- A. $(-2, 0)$ and $(\frac{1}{2}, 0)$
- B. $(2, 0)$ and $(-\frac{1}{2}, 0)$
- C. $(3, 0)$ and $(-2, 0)$ and $(\frac{1}{2}, 0)$
- D. $(3, 0)$ and $(2, 0)$ and $(-\frac{1}{2}, 0)$
- E. $(0, 0)$ and $(2, 0)$ and $(-\frac{1}{2}, 0)$



8. How many solutions does the following system of equations have?

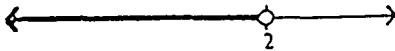
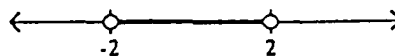
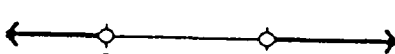
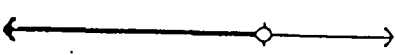
$$\begin{cases} x^2 + y^2 = 20 \\ y^2 - x^2 = 12 \end{cases}$$

- A. 4
- B. 3
- C. 2
- D. 1
- E. 0

9. In a Cartesian coordinate system, what is the equation of the straight line passing through the point $(0, -5)$ and parallel to the straight line whose equation is $y = 2x + 3$.

A. $x + 2y + 5 = 0$
B. $2x - y - 5 = 0$
C. $2x + 3 = -5$
D. $2x - 5y + 3 = 0$
E. $2x + y + 5 = 0$

10. Which interval on the x -axis describes the solution to $x^2 < 4$?

A. 
B. 
C. 
D. 
E. None of the above

11. Two mathematical models are proposed to predict the return y , in dollars, from the sale of x thousand units of an article (where $0 < x < 5$). Each of these models, A and B, is based on different marketing methods.

model A $y = 6x - x^2$

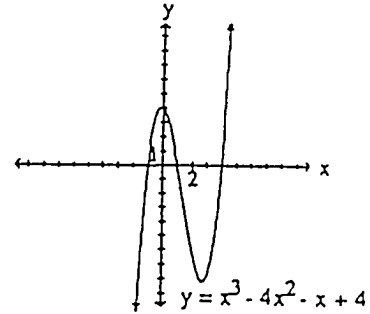
model B $y = 2x$

For what values of x does model B predict a greater return than model A?

A. $0 < x < 4$
B. $0 < x < 5$
C. $3 < x < 5$
D. $3 < x < 4$
E. $4 < x < 5$

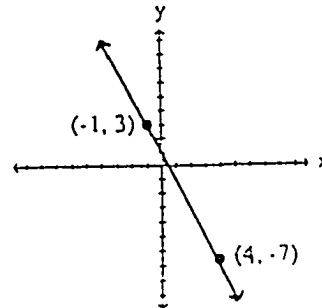
12. Which of the following $(x - 1)$, $(x - 2)$, $(x + 2)$, $(x - 4)$ are factors of $x^3 - 4x^2 - x + 4$?

- A. Only $(x - 1)$
- B. Only $(x - 1)$ and $(x + 2)$
- C. Only $(x - 2)$ and $(x + 2)$
- D. Only $(x + 2)$ and $(x - 4)$
- E. Only $(x - 1)$ and $(x - 4)$



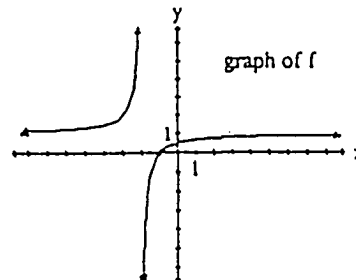
13. The slope of the line through the two points $(-1, 3)$ and $(4, -7)$ is

- A. $-\frac{1}{2}$
- B. $-\frac{3}{4}$
- C. $-\frac{4}{3}$
- D. -2
- E. $-\frac{10}{3}$



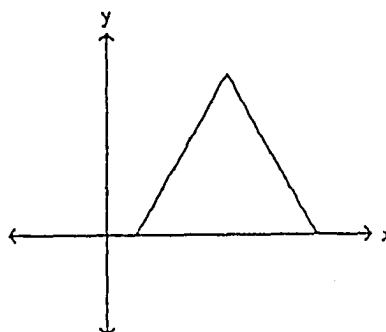
14. Let $f(x) = \frac{x}{x+2}$. What happens to the functional values $f(x)$ as x increases without bound in the positive direction?

- A. $f(x)$ increases toward 1.
- B. $f(x)$ decreases toward 1.
- C. $f(x)$ increases toward $\frac{1}{2}$.
- D. $f(x)$ decreases toward $\frac{1}{2}$.
- E. $f(x)$ decreases toward 0.



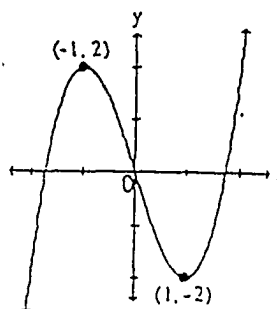
15. One side of an equilateral triangle lies along the x-axis. The sum of the slopes of the three sides is

- A. 0
- B. -1
- C. 1
- D. $2\sqrt{3}$
- E. $1 + 2\sqrt{3}$



16. The diagram shows the sketch of the graph of the cubic function f . The function f could only be given by $f(x)$ is equal to

- A. $-x^3 - x$
- B. $x^3 - 3x^2$
- C. $x^3 - 3x$
- D. $3x^3 - x$
- E. $x^3 + 3x^2$



17. The functions f and g are defined by $f(x) = x - 1$ and $g(x) = (x + 3)^2$. $g(f(x))$ is equal to

- A. $(x-1)(x + 3)^2$
- B. $(x + 3)^2 - 1$
- C. $(2x - 2)^2$
- D. $(x + 2)^2$
- E. $x^2 + 8$

18. If $\log N = n$, then $\log N^2$ is equal to

- A. $n + 2$
- B. n^2
- C. $\frac{n}{2}$
- D. $2n$
- E. $n - 2$

19. The function f , defined by $f(x) = \frac{(x - 1)(3x + 1)}{(2x - 1)(x - 2)}$, is negative for all x such that

- A. $-\frac{1}{3} < x < 3$
- B. $\frac{1}{2} < x < 2$
- C. $1 < x < 3$
- D. $\frac{1}{2} < x < 2$ or $2 < x < 3$
- E. $-\frac{1}{3} < x < \frac{1}{2}$ or $1 < x < 2$

20. If the graph of $y = f(x)$ passes through the origin $(0, 0)$, then the graph of $y = f(x - h) - k$ must pass through

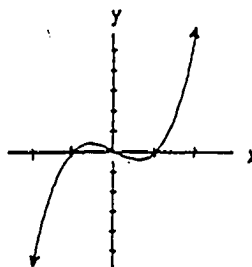
- A. $(0, 0)$
- B. (h, k)
- C. $(h, -k)$
- D. $(-h, k)$
- E. $(-h, -k)$

21. Which of the following pairs of angles can be used as a COUNTEREXAMPLE to show that $\sin(A + B) = \sin A + \sin B$ is not always true?

- A. $A = 30^\circ$, $B = -30^\circ$
- B. $A = 180^\circ$, $B = 180^\circ$
- C. $A = 90^\circ$, $B = 360^\circ$
- D. $A = 90^\circ$, $B = 270^\circ$
- E. $A = 90^\circ$, $B = 90^\circ$

22. The graph displayed to the right can best be represented by which one of the following functions?

- A. $f(x) = x^2 - x - 10$
- B. $f(x) = x^3$
- C. $f(x) = x^3 - x$
- D. $f(x) = x^3 + x^2 + x$
- E. $f(x) = x^3 + x^2 + x - 10$

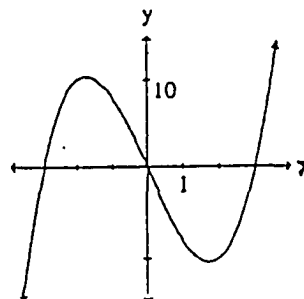


23. For which of the following values of m is the graph of $y^2 - 2y + mx^2 + (2m + 1)x = 0$ a parabola?

- A. $m = -\frac{1}{2}$
- B. $m = 0$
- C. all values of m except $-\frac{1}{2}$
- D. all values of m except 0
- E. all values of m except $-\frac{1}{2}$ and 0

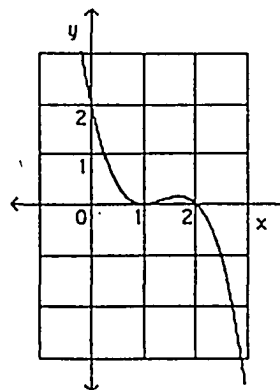
24. This graph is increasing on which intervals?

- A. $(-\infty, -10]$ and $[10, \infty)$
- B. $[-10, -3]$ and $[3, 10]$
- C. $(-\infty, 0]$ and $[3, \infty)$
- D. $[-3, -1.7]$ and $[1.7, 3]$
- E. $(-\infty, -1.7]$ and $[1.7, \infty)$



25. This graph is the representation of one of the following equations. Which one does it represent?

- A. $y = (1 - x)(x - 2)$
- B. $y = (1 - x)(2 - x)$
- C. $y = (1 - x)((2 - x)^2)$
- D. $y = (1 - x)^2(x - 2)$
- E. $y = (1 - x)^2(2 - x)$



26. The domain of $g(x) = \sqrt{6 - 3x}$ is

- A. All real numbers
- B. $x \leq 2$
- C. $x \geq 0$
- D. $x \geq 2$
- E. None of these

27. $\frac{\cot x - 1}{1 - \tan x} =$

- A. $\sin x$
- B. $\cos x$
- C. $\tan x$
- D. $\csc x$
- E. $\cot x$

28. The function $f(x) = \tan x$ is *not defined* for

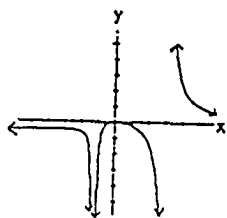
- A. $x = \frac{-\pi}{2}$
- B. $x = 0$
- C. $x = \frac{\pi}{4}$
- D. $x = \pi$
- E. None of these

29. One root of $3x^2 - 2x - 4 = 0$ is

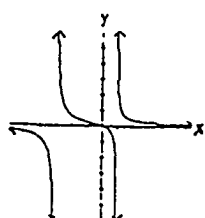
- A. $\frac{1 - 2\sqrt{13}}{3}$
- B. $\frac{-2 - \sqrt{13}}{3}$
- C. $-\frac{2}{3}$
- D. $\frac{1 + \sqrt{13}}{3}$
- E. None of these

30. The graph of $\frac{x}{x^2 - 2x - 3}$ is best represented by:

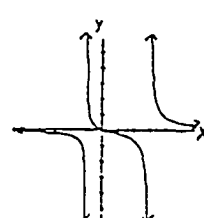
A)



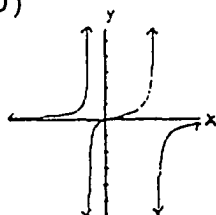
B)



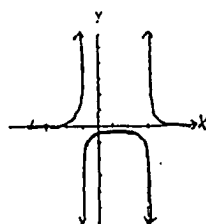
C)



D)



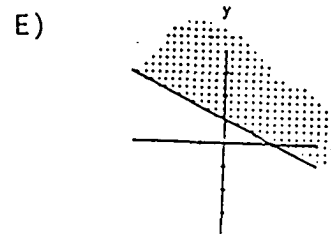
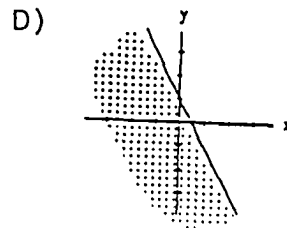
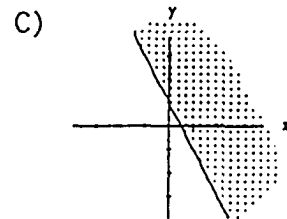
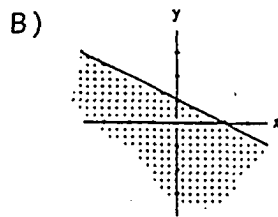
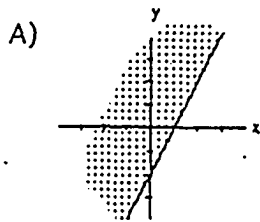
E)



31. If $\sin x = \frac{5}{13}$ and x is acute then $\sec x$ is

- A. $\frac{12}{13}$
- B. $\frac{13}{12}$
- C. $\frac{12}{5}$
- D. $\frac{13}{5}$
- E. None of these

32. The graph of $2x + y \leq 1$ is best represented by:

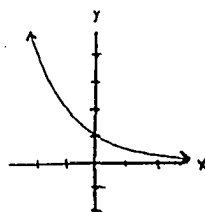


33. Solve $\log_x 289 = 4$ for x

- A. $(289)^{1/4}$
- B. $(289)^4$
- C. 4^{-289}
- D. 4^{289}
- E. None of these

34. Which of the following equations is best represented by the graph to the right?

- A. $y = \sqrt{x}$
- B. $y = \frac{1}{\sqrt{x}}$
- C. $y = \log_2 x$
- D. $y = 2^x$
- E. $y = 2^{-x}$



35. Solve for x : $|x - 1| > 3$

- A. $-4 < x < 2$
- B. $x > 4$ or $x < -2$
- C. $-2 > x > 4$
- D. $x > 4$
- E. $-2 < x < 4$

APPENDIX C ATTITUDE SCALE

Mathematics Attitudes

This survey asks how you feel about mathematics or mathematics activities. No answers are incorrect. The answer choices are: Strongly Agree, Agree, Undecided, Disagree, or Strongly Disagree.

A. USE A #2 PENCIL TO ENTER YOUR SOCIAL SECURITY NUMBER IN THE IDENTIFICATION NUMBER GRID ON THE ANSWER SHEET. DO NOT PUT YOUR NAME ON YOUR ANSWER SHEET. Identification is needed for matching purposes only. This information will be used for purposes of the study only and will have no impact on your grade in this class.

For each statement, choose the one response that best describes how you feel about the statement. Be sure to fill in one choice on the answer sheet for each statement.

Strongly Agree	Agree	Undecided	Disagree	Strongly Disagree
A	B	C	D	E

1. I am good at mathematics.
2. It is important to know mathematics such as algebra and geometry in order to get a good job.
3. It is important to know arithmetic to get a good job.
4. I am taking mathematics only because I have to.
5. New discoveries are seldom made in mathematics.
6. Mathematics is more for males than females.
7. Creative people usually have trouble with mathematics.
8. Estimating is an important mathematical skill.
9. I usually understand what we are talking about in mathematics.
10. Most of mathematics has practical value.
11. Knowing how to solve a problem is as important as getting the solution.
12. Doing mathematics requires lots of practice in following rules.
13. I can get along well in everyday life without using mathematics.
14. Mathematicians work with symbols rather than ideas.
15. Fewer men than women have the logical ability to become mathematicians.

16. Knowing why an answer is correct is as important as getting the correct answer.

Strongly Agree	Agree	Undecided	Disagree	Strongly Disagree
A	B	C	D	E

17. Mathematics is made up of unrelated topics.
18. I really want to do well in mathematics.
19. I feel good when I solve a mathematics problem by myself.
20. Solving word problems is more fun if you use a calculator.
21. Guess and check can be used to solve a mathematics problems.
22. Using a calculator can help you learn many different mathematical topics.
23. Mathematics is more for females than for males.
24. Learning mathematics is mostly memorizing.
25. Mathematics is useful in solving everyday problems.
26. Exploring number patterns plays almost no part in mathematics.
27. There is always a rule to follow in solving mathematics problems.
28. A good grade in mathematics is important to me.
29. Justifying the mathematical statements a person makes is an extremely important part of mathematics.
30. A mathematics problem can always be solved in different ways.
31. I am good at working with numbers.
32. I enjoy going beyond the assigned work and trying to solve new problems in mathematics.
33. Mathematics is enjoyable and stimulating to me.
34. Mathematics makes me feel uneasy and confused.
35. I am interested and willing to use mathematics outside of school and on the job.
36. I have never liked mathematics, and it is my most dreaded subject.
37. I have always enjoyed studying mathematics in school.
38. I would like to develop my mathematical skills.
39. Mathematics makes me feel uncomfortable and nervous.

40. I am interested and willing to acquire further knowledge of mathematics.
41. Mathematics is dull and boring because it leaves no room for personal opinion.

Strongly Agree	Agree	Undecided	Disagree	Strongly Disagree
A	B	C	D	E

42. Mathematics is very interesting, and I have usually enjoyed courses in this subject.
43. Mathematics has contributed greatly to science and other fields of knowledge.
44. Mathematics is less important to people than art or literature.
45. Mathematics is not important for the advance of civilization and society.
46. Mathematics is a very worthwhile and necessary subject.
47. An understanding of mathematics is needed by artists and writers as well as scientists.
48. Mathematics helps develop a person's mind and teaches one to think.
49. Mathematics is not important in everyday life.
50. Mathematics is needed in designing practically everything.
51. Mathematics is needed in order to keep the world running.
52. There is nothing creative about mathematics; it's just memorizing formulas and things.

APPENDIX D MATHEMATICS STUDY PROCESS INSTRUMENT

133
Mathematics Study Process

Directions:

To help me understand your study habits and class attendance practices, I will ask for the following information several times throughout the semester. This information will be used for purposes of the study only and will have no impact on your grade in this class.

A. USE A #2 PENCIL TO ENTER YOUR SOCIAL SECURITY NUMBER IN THE IDENTIFICATION NUMBER GRID ON THE ANSWER SHEET. DO NOT PUT YOUR NAME ON YOUR ANSWER SHEET. Identification is needed for matching purposes only.

1. In the last week (4 classes), I have attended class ___ times.

- A) 4 D) 1
B) 3 E) 0
C) 2

2. If any classes were missed, mark the major reason:

- A) illness
B) university athletics
C) pressure of test or paper due for another class
D) overslept
E) class doesn't help me learn
F) I am too far behind to understand what is being discussed
G) class is boring
H) job
I) other

3. During the last week, did you attend Math Lab?

- A) Yes --> Go to #4
B) No --> Go to #16

4 -15. Think about how you spend your time in Math Lab. For each activity listed, choose the letter from the table below that best represents the portion of time you spend on that activity in Math Lab.

All my time	Most of my time	Some time	A little time	No time
A	B	C	D	E

4. Work my homework problems.
5. Check the answers to my homework problems.
6. Ask questions about my homework problems.
7. Study the instructor's lecture notes.
8. Ask questions about mistakes on previous exams or quizzes.
9. Study the quiz or test answer keys.
10. Read the textbook.
11. Review my class notes.
12. Rework previously missed problems.
13. Ask questions other than on homework or test problems.
14. Think about the ideas we are studying.
15. Solve additional unassigned problems.
16. Do you study mathematics outside of Math Lab?
 - A) Yes --> Go to #17
 - B) No --> Go to #29.

17 - 24. Think about how you spend your time when you study mathematics outside of Math Lab. For each activity listed, choose the letter from the table below that best represents the portion of time you spend on that activity outside of Math Lab.

All my time	Most of my time	Some time	A little time	No time
A	B	C	D	E

17. Read the textbook.
18. Review my class notes.
19. Work my homework problems
20. Study mistakes from previous homework assignments or tests.
21. Solve additional unassigned problems.
22. Think about the ideas we are studying.
23. Rework previously missed problems.
24. Check the answers to my homework problems at the library.

25 - 28. Think about whether you study mathematics alone or with others outside of Math Lab. For each activity listed, choose the letter from the table below that best represents the portion of time you spend on that activity outside of Math Lab.

All my time	Most of my time	Some time	A little time	No time
A	B	C	D	E

25. Study by myself.
 26. Study with others.
 27. Attend my Supplemental Instruction (SI) group meeting.
 28. Visit my instructor to ask questions.
29. Approximate length of time since I finished my last math course:
 A) 0 - 4 months
 B) 5 - 10 months
 C) 11 - 15 months
 D) 1.5 years ago
 E) 2 years ago
 F) 2.5 years ago
 G) 3 years ago
 H) 3.5 - 5 years ago
 I) 5.5 - 10 years ago
 J) more than 10 years ago
- 30 - 37. Math courses taken in high school:
 Mark answers as (A) YES or (B) NO
 30. First year Algebra (or its equivalent)
 31. Geometry
 32. Second year Algebra (or its equivalent)
 33. Trigonometry
 34. Precalculus or College Algebra
 35. Discrete Math
 36. Probability or Statistics
 37. Calculus

38 - 45. Math courses taken in college:

Mark answers as (A) YES or (B) NO

- 38. First year Algebra (or its equivalent)
- 39. Geometry
- 40. Second year Algebra (or its equivalent)
- 41. Trigonometry
- 42. Precalculus or College Algebra
- 43. Discrete Math
- 44. Probability or Statistics
- 45. Calculus

46. Have you used a calculator before? A) Yes B) No

47.- 49. What type(s) of calculator have you used?

Mark answers as (A) YES or (B) NO

- 47. four function calculator (+, -, x, ÷)
- 48. scientific calculator (+, -, x, ÷, y^x , log, trig functions)
- 49. graphing calculator

50. Before entering this course, how competent were you with a graphing calculator?

- A) very competent
- B) somewhat competent
- C) some knowledge and experience
- D) little knowledge and experience
- E) no experience with a graphing calculator

51. Reason for taking this course (mark only the main reason)

- A) satisfies general education requirement
- B) required for my major
- C) prerequisite for another course I will take
- D) an elective that I chose

52. Social Security Number: ____ - ____ - ____

In the table below, put an X in the box for each half hour or part thereof that you spent studying math for this class during the last week. (Include time in Math Lab, attending a Supplemental Instruction meeting, and any other time studying on your own as well).

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
7:00 AM							
7:30 AM							
8:00 AM							
8:30 AM							
9:00 AM							
9:30 AM							
10:00 AM							
10:30 AM							
11:00 AM							
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11:30 PM							
12:00 AM							
12:30 AM							
1:00 AM							

1:30 AM							
2:00 AM							
2:30 AM							

53. In the tables below, put an "X" in each half hour time period or part thereof that you spent in Math Lab during the past week.

Day Math Lab:

	Tuesday	Thursday
9 a.m.		
9:30 a.m.		
10 a.m.		
10:30 a.m.		
11 a.m.		
11:30 a.m.		
12:00 p.m.		
12:30 p.m.		
1:00 p.m.		
1:30 p.m.		
2:00 p.m.		
2:30 p.m.		
3:00 p.m.		
3:30 p.m.		
4:00 p.m.		

Night Math Lab:

	Monday	Tuesday	Wednes.	Thurs.		Sunday
9:00 p.m.					7:00 p.m.	
9:30 p.m.					7:30 p.m.	
10 p.m.					8:00 p.m.	

APPENDIX E FACTOR ANALYSIS OF ATTITUDE SCALE

Factor Analysis of the Attitude Scale

A factor analysis using the principal components extraction method and varimax rotation was used to identify the main factors using pretest scores on the attitude instrument. Two factors were identified. The first factor, which is similar to Aiken's Enjoyment scale, also appears to measure enjoyment of mathematics. The reliability for the twelve items in the enjoyment factor was .92. The items loading on this component are shown below:

Factor 1: Factor analysis enjoyment of mathematics

1. I am good at mathematics.
4. I am taking mathematics only because I have to.
9. I usually understand what we are talking about in mathematics.
31. I am good at working with numbers.
32. I enjoy going beyond the assigned work and trying to solve new problems in mathematics.
33. Mathematics is enjoyable and stimulating to me.
34. Mathematics makes me feel uneasy and confused.
36. I have never liked mathematics, and it is my most dreaded subject.
37. I have always enjoyed studying mathematics in school.
39. Mathematics makes me feel uncomfortable and nervous.
42. Mathematics is very interesting, and I have usually enjoyed courses in this subject.
52. There is nothing creative about mathematics; it's just memorizing formulas and things.

The second factor which was identified in the factor analysis is similar to Aiken's Value scale, containing many of the same items. It also appeared to measure the student's attitude toward the value of mathematics; the reliability for the twelve items in this factor was 0.85.

Factor 2: Factor analysis value of mathematics

2. It is important to know mathematics such as algebra and geometry in order to get a good job.
13. I can get along well in everyday life without using mathematics.
14. Mathematicians work with symbols rather than ideas.
25. Mathematics is useful in solving everyday problems.
35. I am interested and willing to use mathematics outside of school and on the job.
45. Mathematics is not important for the advance of civilization and society.
46. Mathematics is a very worthwhile and necessary subject.
47. An understanding of mathematics is needed by artists and writers as well as scientists.
48. Mathematics helps develop a person's mind and teaches one to think.
49. Mathematics is not important in everyday life.
50. Mathematics is needed in designing practically everything.
51. Mathematics is needed in order to keep the world running.

The researcher examined the enjoyment and value scales that were identified using factor analysis of the attitude instrument. An analysis of variance of posttest factor analysis enjoyment of mathematics scores by treatment and teacher with factor analysis enjoyment of mathematics scale pretest scores as covariate was performed. Shown below in Table 18 are the means and standard deviations

of the pretest and posttest scores of the four classes for the enjoyment scale items determined by the factor analysis.

The analysis of variance of posttest factor analysis enjoyment of mathematics scale scores by treatment and teacher with factor analysis enjoyment of mathematics scale pretest scores as covariate did not show a significant difference due to treatment as shown in Table 19. Also, the analysis did not show a significant difference in enjoyment of mathematics attitudes due to teacher nor a significant difference due to treatment x teacher interaction.

Table 18. Means and standard deviations: factor analysis enjoyment scale pretest and factor analysis enjoyment scale posttest scores for each class

Treatment	Writing	Nonwriting	Writing	Nonwriting	Total
Teacher	1	1	2	2	
Pretest					
N	52	51	52	54	209
Mean	34.77	35.08	35.15	36.35	35.35
StdDev	8.93	9.86	9.38	10.75	9.71
Possible					
Range	12 - 60	12 - 60	12 - 60	12 - 60	12 - 60
Posttest					
N	52	51	52	54	209
Mean	36.23	34.61	35.67	36.69	35.81
StdDev	8.79	9.35	10.30	10.18	9.64
Possible					
Range	12 - 60	12 - 60	12 - 60	12 - 60	12 - 60

Table 19. Analysis of variance of posttest factor analysis enjoyment scale score by treatment and teacher with pretest factor analysis enjoyment scale score as covariate

Source of Variation	Sum of Squares	df	Mean Square	F	Sig. of F
Covariate					
Achpre	13063.21	1	13063.21	430.54	.00
Main Effects	43.29	2	21.64	.71	.49
treatment	43.00	1	43.00	1.42	.24
teacher	.39	1	.39	.01	.91
Interaction	47.58	1	47.58	1.57	.21
Residual	6189.65	204	30.34		

An analysis of variance of posttest factor analysis value of mathematics scale scores by treatment and teacher with factor analysis value of mathematics scale pretest scores as covariate was performed. Shown below in Table 20 are the means and standard deviations of the pretest and posttest scores of the four classes for the value scale items determined by the factor analysis.

The analysis of variance of posttest factor analysis value of mathematics scale scores by treatment and teacher with factor analysis value of mathematics scale pretest scores as covariate did not show a significant difference due to treatment as shown in Table 21. Also, the analysis did not show a significant difference in value of mathematics attitudes due to teacher nor a significant difference due to treatment x teacher interaction. Finally, the analysis of variance of the posttest attitude scores on the items identified by the factor analysis did not show a significant difference for the treatment groups.

Table 20. Means and standard deviations: factor analysis value scale pretest and factor analysis value scale posttest scores for each class

Treatment Teacher	Writing 1	Nonwriting 1	Writing 2	Nonwriting 2	Total
Pretest					
N	52	51	52	54	209
Mean	45.65	45.16	45.09	45.17	45.27
StdDev	6.43	6.10	6.15	7.00	6.39
Possible Range	12 - 60	12 - 60	12 - 60	12 - 60	12 - 60
Posttest					
N	52	51	52	54	209
Mean	46.79	44.92	46.04	46.17	45.99
StdDev	5.61	6.50	7.03	6.11	6.32
Possible Range	12 - 60	12 - 60	12 - 60	12 - 60	12 - 60

Table 21. Analysis of variance of posttest factor analysis value scale score by treatment and teacher with pretest factor analysis value scale score as covariate

Source of Variation	Sum of Squares	df	Mean Square	F	Sig. of F
Covariate					
Achpre	3751.79	1	3751.79	170.43	.00
Main Effects	36.14	2	18.07	.82	.44
treatment	26.79	1	26.79	1.22	.27
teacher	9.79	1	9.79	.45	.51
Interaction	34.21	1	34.21	1.55	.21
Residual	4490.82	204	22.01		